AN OPTIMUM WEIGHT DESIGN METHOD FOR LONGITUDINALLY STIFFENED PLATES SUBJECTED TO COMBINED AXIAL AND LATERAL LOADS

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ABSTRACT

The object of this report is to develop an optimum weight design method for longitudinally stiffened plates subjected to combined axial and normal loads. Although the development of this method is general, the specific case investigated is a flat rectangular plate, with equally spaced stiffeners oriented in the direction of the axial compressive stress. The lateral load is taken to be uniformly distributed, acting normal to the plane of the plate. The loaded edges of this stiffened panel are assumed to be simply supported; no account is taken of the support of the unloaded edges of the overall panel.

The analysis is carried out on the assumption that the failure is of an instability type. The optimum weight criterion used is that the buckling strength of the stiffeners and the local plate panels between stiffeners should be equal. The strength of the stiffener is treated through the use of a simple interaction formula incorporating the tangent-modulus column buckling equation; a suitable effective width of plating is assumed to act with the stiffener. The plate buckling is treated through the use of Bryan's equation for plate buckling modified by Bleich to take account of inelastic behavior. Limitations of the assumption of an instability failure are discussed.

Although the analysis is general, numerical results are worked out for the case of T-type stiffeners in combination with standard plate thicknesses for a range of loadings and geometry applicable to ship-type structures. Direct-reading design charts are developed for a few specific cases using mild steel as a material. These design charts enable the designer to select the proper stiffener spacing, size of stiffener, and plate thickness to satisfy the optimum weight condition, providing the designer knows the axial design stress (or load), the equivalent head of salt water causing the uniformly distributed lateral load, and the panel length.

The specific cases investigated cover the range of plate thicknesses from 1/4 inch to 3/4 inch; length of panel from 8 ft. to 16 ft.; and heads from about 0 to 40 ft.

In the range considered it was found that the optimum geometry for this type of panel generally consists of thin plating and narrow frame spacing.

It is concluded that the design method developed in this report is practical and simple to use. The method of analyzing the structure, although approximate, is adequate for engineering work.

It is recommended that curves of the nature shown in this paper be derived to cover more completely the range of interest in ship structural design, and that these curves actually be used in design of such elements of the ship as decks and bottom plating.

Thesis supervisor: J. Harvey Evans

Title: Associate Professor of Naval Architecture

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NOTATION

a = length of the panel or stiffeners in the direction of the applied load. (Inches unless otherwise noted.)

b = stiffener spacing (In.)

b_e = effective breadth of plating acting with the stiffener in bending (In.)

f = plate deflection (In.)

p = uniformly distributed lateral pressure load (lbs/in.^{2})

r = radius of gyration of plate-stiffener combination (In.)

t = plate thickness (In.)

w_e = effective width of plating acting with the stiffener in considering column-type instability.

A_s = cross sectional area of a stiffener element--with no effective plating (In.^{2})

E = Young's Modulus

E_t = tangent modulus

F.S.= factor of safety

H = head of fluid causing uniformly distributed normal pressure on the panel. (ft.)

H_p = head acting on the smooth side of the plate (ft.)

H_s = head acting on the stiffener side of the panel. (ft.)

K_b = constant in the expression for bending stress--a function of the boundary conditions.

K_c = constant in the expression for column critical stress--a function of the boundary conditions.

K_p = constant in the expression for plate critical stress--a function of boundary conditions.

N = nominal axial compressive load per unit breadth of panel. $N = (A_s + bt)(\sigma_{a/b})$ (lbs./in.)

P = critical buckling load (lbs.)

Z = section modulus of plate-stiffener combination. (In.^{3})

Z_f = section modulus of plate-stiffener combination using extreme fiber distance to the flange.

Z_p = section modulus of plate-stiffener combination using extreme fiber distance to the plate.

γ = specific weight

γ for salt water: 64 lbs./ft.^{3}

γ for steel: .283 lbs/in.^{3}

σ = stress (lbs./in.^{2})

$\tau = \frac{E_t}{E}$

μ = Poisson's ratio (for steel μ = .3)

Subscripts:

a .... refers to applied or actual.

all ..... refers to allowable.

b .... refers to bending.

c .... refers to compression.

col ..... refers to column.

cr ...... refers to critical.

des ..... refers to design.

f .... refers to flange.

p .... refers to plate.

s .... refers to stiffener.

y .... refers to yield.

ACKNOWLEDGMENT

The authors wish to express their appreciation to Professor J. Harvey Evans of the Department of Naval Architecture and Marine Engineering, their thesis supervisor, who initially aroused their interest in the area of minimum weight design and who gave his time generously in assisting them and encouraging them over several obstacles.

I. INTRODUCTION

The subject of minimum weight analysis in compression structures has been of interest in the field of aircraft structural design for several years, and as a result there are a substantial number of strength-weight investigations, both theoretical and experimental, of typical aircraft structural elements. Furthermore, successful application of optimum weight design methods in the field of aircraft structural design has aroused a general interest in possible applications to structures in other fields, including the field of ship structural design where there are many applications for which a definite benefit could be obtained from the use of minimum weight design procedures. With the exception of rather general methods used, most of the work done in the aircraft field applies to structures which differ too much from common structural elements of ships to have any direct applicability in the area of ship structural design.

The only work known to the authors done along lines applicable to ship structures is a thesis by Harlander (1) who treats two cases--a stiffened plate subject to lateral

load, and a longitudinally stiffened plate subjected to axial compressive loads. It is also noted that considerable work has been done in the aircraft field on the minimum weight analysis of longitudinally stiffened plates subjected to axial compressive loads.

A case which is common in ship structural design is that of a longitudinally stiffened plate subjected to combined axial compressive loads and lateral bending loads. (A typical example of this problem in ship structures is the deck or bottom plating of a longitudinally framed ship. The axial loading would be the ship bending stress; the lateral load would be a deck load or hydrostatic sea pressure.)

The purpose of this report is to develop a practical method of analyzing this type of structure and loading on a weight--strength basis, for a range of geometries typical to ship-type structures, and to develop the method of analysis into a suitable design procedure. Generally accepted strength formulations are used and a minimum weight criterion is applied to obtain a straightforward design method. Standard plates and stiffeners are used to obtain numerical results which are presented in the form of design charts.

II. PROCEDURE

Geometry, Loading and Boundary Conditions

Under consideration is a flat longitudinally stiffened panel, rectangular in shape, with dimensions as shown in Figure 1. The stiffeners divide the overall gross panel into local plate panels. Although there is no restriction on the ratio of the length to width of the overall panel, it is assumed that the aspect ratio of the local plate panels, a/b, is greater than 2.

The panel is subjected to combined uniform axial compressive stress (equal in the plate and stiffener) and uniformly distributed lateral load (normal to the plane of the panel). The lateral load may act on either side of the panel, but it is important in the analysis which side is considered loaded.

The overall panel is supported along its loaded edges by simple supports. The unloaded edges are assumed to be unsupported although the support of these edges has little effect on the analysis for very wide panels at locations distant from the vicinity of the unloaded edges.

Analysis

Type of Failure. It is assumed that the primary failure of the panel is of the instability type. Four types of instability or buckling failure are possible in the stiffened panel:

(1) Local instability of one or more simple elements of the stiffener.

(2) Torsional instability of the stiffener.

(3) Local instability of the plate alone.

(4) General instability of the panel involving column action of the stiffeners.

This analysis will apply to so-called sturdy stiffeners, i.e., those whose dimensions are so proportioned that they withstand torsional buckling and local failure or else are suitably supported by intermediate bracing to prevent these failures. Therefore modes (1) and (2) are not possible under the assumptions of this analysis.

The Optimum Weight Condition. The optimum distribution of material for maximum buckling strength will occur when local buckling of the plate and general instability of the panel, through column-type failure of the stiffeners, occur simultaneously.[1] This criterion may be justified

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1 It should be noted that simultaneous occurrence of local plate buckling and primary buckling of the stiffened panel does not necessarily ensure the optimum design if the design is based on ultimate load instead of the buckling load.

by simple logic. Assuming that from the designer's standpoint buckling of any element of the structure is as disastrous as primary buckling, if any element has a greater buckling strength than another, some material could be transferred from the stronger element to the weaker element to raise the buckling strength of the weaker element. This redistribution of material may be repeated until all elements of the structure have the same buckling strength, with the result that the buckling strength of any element is greater than the strength of the element which was originally lowest. Or in other words, the buckling strength of the structure has been raised with no increase in material.

Panel Action. In this analysis we are assuming that the panel's primary strength is governed by column-type action. Lundquist (2) has shown a method based on this assumption which agrees well with experimental results for computing the buckling load of a stiffened panel subjected to axial compression only. In essence his approximation treats a single stiffener together with some effective width of plating as a column which fails by bending normal to the plane of the plate. His method is a logical development from the remarks of Von Karman (3) on the strength of plates in com-

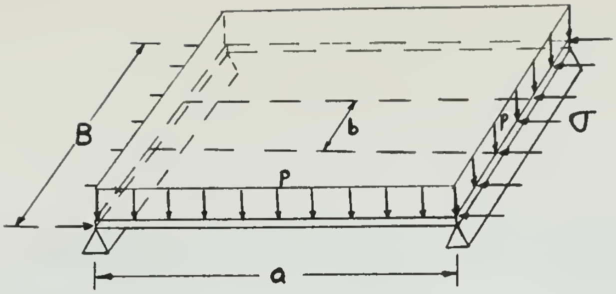

FIGURE 1 - Geometry of Panel.

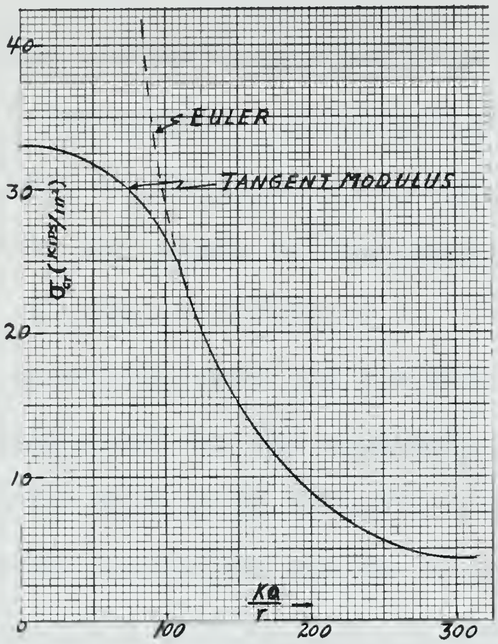

FIGURE 2 - Column Critical Stress.

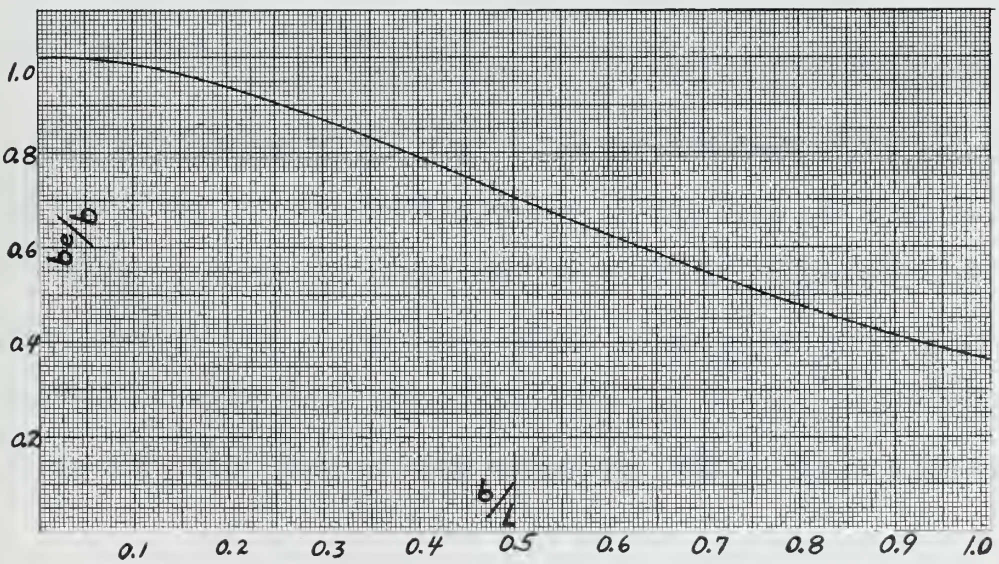

FIGURE 3 - Vedeler's Effective Breadth of Plating in Bending.

pression, and it has been used extensively in the aeronautical field of structures.

Under the action of combined loading it is proposed to treat the stiffener and some effective amount of plating as a beam-column which fails by bending normal to the plane of the panel. (This assumes that sufficient bracing is present to preclude stiffener buckling in the plane parallel to the plane of the panel if that is the weak direction of the stiffener.)

Using the proposition that the primary strength of the panel is equivalent to the strength of a single stiffener together with some effective amount of plating, the optimum weight criterion may be slightly restated as follows: <u>The optimum distribution of material for maximum buckling strength will occur when local buckling of the plate occurs simultaneously with buckling of a stiffener-plate combination treated as a beam-column.</u>

<u>The Stiffener Stability</u>. The most commonly used method of handling the beam-column is by the use of interaction formulas. Several types of interaction formulas are in use, and it has been shown that in general they give good agreement with both experimental and more exact theoretical analyses. (4) One of the simplest and perhaps the most

widely used interaction formulas applied to the beam-column problem is the interaction formula:

$$\frac{\sigma_a}{\sigma_{cr\ col}} \times \frac{\sigma_b}{\sigma_{b\ all}} = 1 \quad \text{Or} \quad \sigma_a = \sigma_{cr\ col}\left[1 - \frac{\sigma_b}{\sigma_{b\ all}}\right] \tag{1}$$

This formula has been compared with experimental and more exact theoretical results with reasonably good agreement. (5) Although more accurate interaction formulas could be accommodated in this analysis, the equation above will be used for two major reasons. It simplifies the ultimate solution of the problem; and since the analysis is only an approximate method, it is felt that more complicated relationships are not warranted.

The following relationships have been used in the interaction formula; they should be self-explanatory:

$$\sigma_{cr\ col} = \frac{\pi^2 E \tau_{col}}{\left(\frac{K_c a}{r}\right)^2} \tag{2}$$

where: K_c = constant determined by the stiffener end conditions.

K_c = 1 for simple end supports.

A plot of the critical stress in equation (2) vs. $\frac{K_c a}{r}$ is shown in Figure 2.

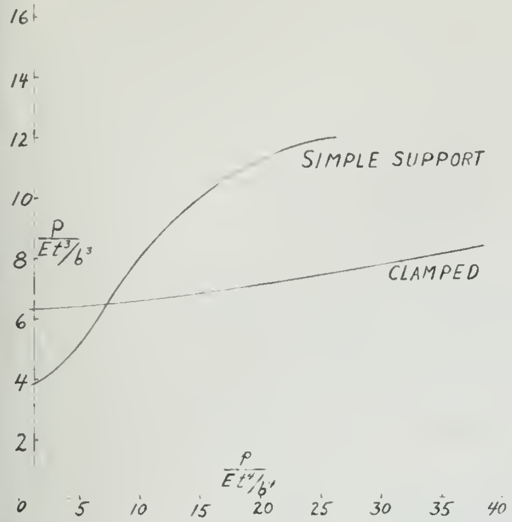

FIGURE 4 - Effect of Lateral Pressure (p) on Plate Critical Load (P) for simply Supported and Clamped Plates.

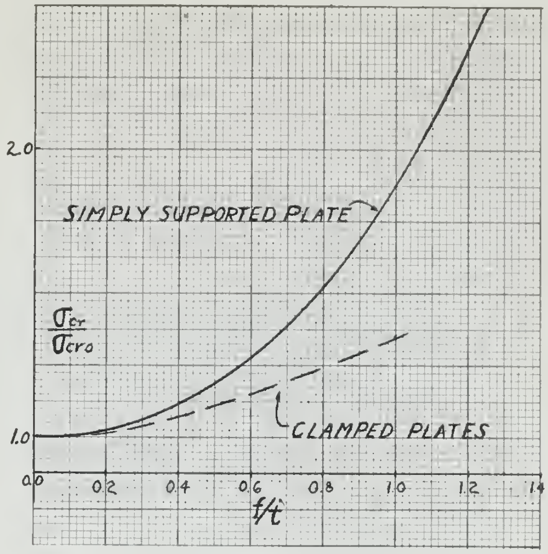

FIGURE 5 - Critical Stress Ratio versus Deflection to Thickness Ratio for Clamped and Simply Supported Plates.

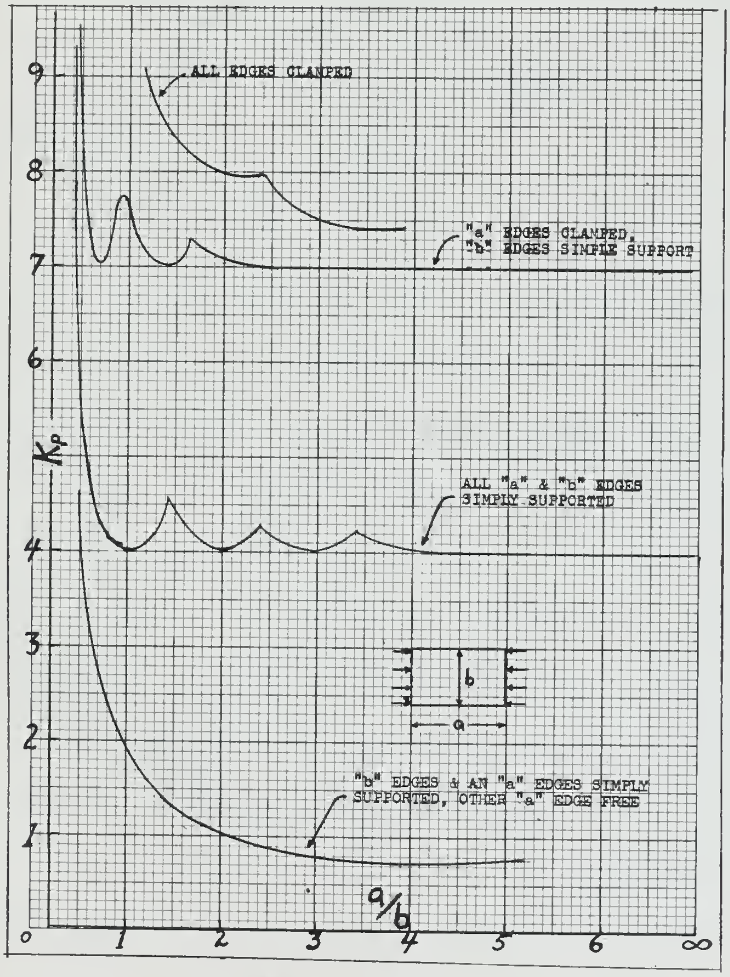

FIGURE 6 - Values of Coefficient K_p in Bryan Formulas for Critical Stress in Panels of Plating Under Edge Loadings.

$$\sigma_b = \frac{pba^2}{K_b Z}$$

where: p force per unit area of the normal load (p.s.i.)

Or for the case of the normal load caused by a head of fluid, the bending stress can be written as:

$$\sigma_b = \frac{\gamma H b\, a^2}{(K_b Z)\, 144} \qquad (3)$$

where: γ is the specific weight of the fluid (lbs./cu.ft.)

H = head of fluid (ft.)

K_b = constant depending on stiffener end fixity conditions.

K_b = 8 for simple end supports.

And for no factor of safety:

$$\sigma_{b\ all} = \sigma_y$$

It must borne in mind that the section modulus, Z, applies to the outermost fiber of the plate-stiffener combination <u>which is in compression due to the bending load</u>. Thus, if the stiffener is not symmetrical about the middle plane of the plate, the correct selection of section modulus is dependent upon which side of the panel is subjected to the lateral loading. For instance, if the stiffeners are on one side of the panel only, and the lateral load is applied to the same side of the panel, the outermost fiber of the stiffener is in compression and the section modulus to be used in Equation (3)

is the modulus which corresponds to the distance between the neutral axis of the plate-stiffener combination and this fiber. If the load is on the smooth side of the panel, then the distance to the outermost plate fiber is implied.

Effective Width and Effective Breadth. It has been mentioned that Lundquist used some effective width of plating in conjunction with the stiffener and treated the combination as a column. He chose to use Von Karman's experimentally determined effective width, $w_e = 1.70\sqrt{\frac{\sigma t}{E}}$, where w_e is the width of effective plating acting with the stiffener.

Von Karman's experimentally determined effective width was obtained in the testing of the ultimate strength of plating, i.e., plating which in fact had already reached the buckling stress before ultimate failure. In this analysis the amount of plating which acts with the stiffener before plate buckling is of concern. Logically it would appear that the entire width of plating between stiffeners would act with the stiffener, and as a matter of fact when Von Karman's theoretical effective width is used, the effective width does in fact equal the spacing between stiffeners.

$$W_e = t \sqrt{\frac{K_p \pi^2 E}{12(1-\mu^2)\sigma}}$$

When $\sigma = \sigma_{crp}$, $W_e = b$

It may be worthwhile to note at this point that in general an increase in effective width will reduce the radius of gyration of the stiffener-plate combination; so the use of the entire spacing between stiffeners as effective plating is actually conservative. See Figure 11 for the variation of radius of gyration with effective width in a typical example.

In this analysis the entire spacing between stiffeners has been used as the effective width of plating for the purpose of evaluating the radius of gyration of the stiffener-plate combination in Equation (2).

The amount of plating acting with the stiffener in bending is different from the effective width which is assumed for stability conditions. The amount of plating acting in bending is usually referred to as an effective breadth of plating. Vedeler's (6) effective breadth has been chosen for use in this analysis; it is commonly expressed as:

$$\frac{b_e}{b} = \frac{1 + \dfrac{\sinh \pi b/\ell}{\pi b/\ell}}{1 + \cosh \pi b/\ell}$$

where: ℓ = the length between zero bending moment in the stiffener. For simple end supports $\ell = a$.

Figure 3 is a plot of Vedeler's effective breadth.

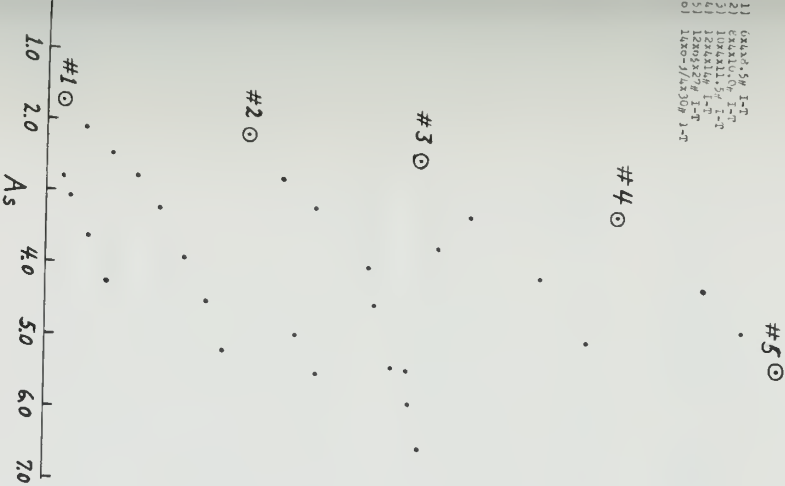

FIGURE 7 - Section Modulus of Plate-Stiffener Combination Using Extreme Fiber Distance to **Plate vs. Stiffener Cross Sectional Aréa for all Standard Tee Sections and 1/2" Plate.**

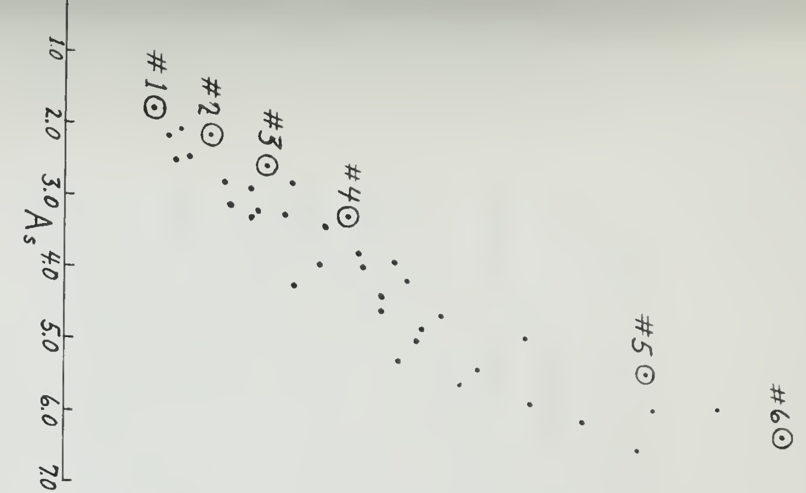

FIGURE 8 - Seetion Modulus of Plate-Stiffener Combination Using Extreme Fiber Distance to Stiffener Flange vs. Stiffener Cross Sectional Area for All Standard Tee Sections and 1/2" Plate.

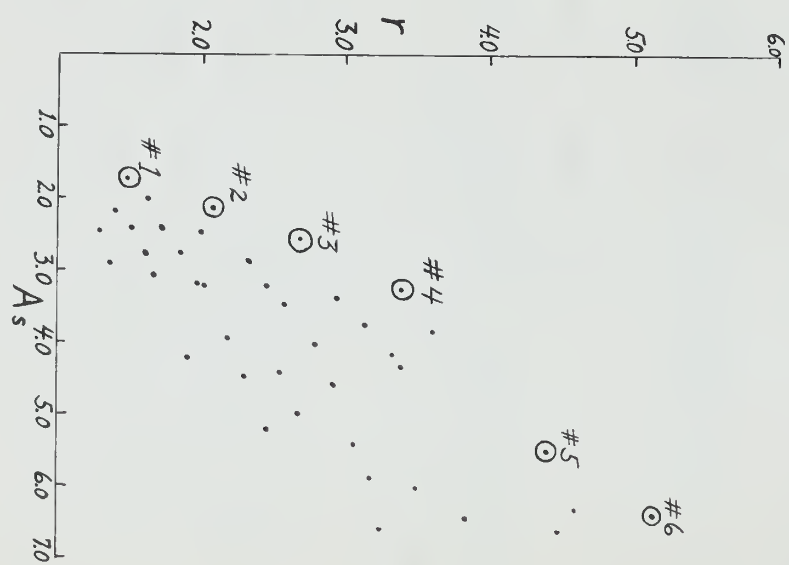

FIGURE 9 - Radius of Gyration of Plate-Stiffener Combination versus Stiffener Cross Sectional Area for All Standard Tee Sections and 1/2" Plate.

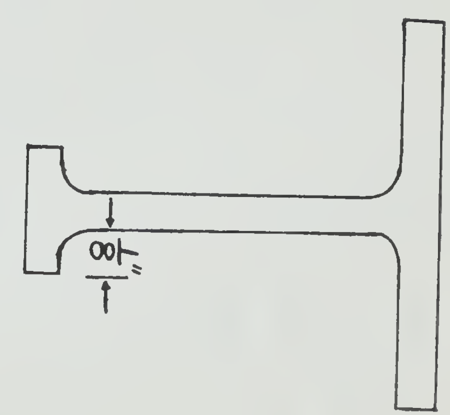

FIGURE 10 - Sketch of I-T Stiffener.

The Local Plate Panel. Limited theoretical solutions in the elastic range have been given for long rectangular plates subjected to combined axial load and normal pressure. For the case of simply supported edges and a length-width ratio of 4:1, Levy et al. (7) have found that the buckling load is considerably increased by normal pressure due to increased membrane stresses in the plate. Their investigation for four normal pressures (which cover the range of interest for ships' structures) yield the following results:

$$P = 3.84 \frac{Et^3}{b} \text{ when } p = 0 \qquad P = 8.56 \frac{Et^3}{b} \text{ when } p = 12.02 \frac{Et^4}{b^4}$$

$$P = 4.05 \frac{Et^3}{b} \text{ when } p = 2.40 \frac{Et^4}{b^4} \qquad P = 11.84 \frac{Et^3}{b} \text{ when } p = 24 \frac{Et^4}{b^4}$$

where: P = critical compressive load (lbs.)

Thus for the highest pressure considered, the theoretical buckling load is 3.1 times the buckling load for zero normal pressure.

For the case of edges clamped and a length-width ratio of 4:1, Corrick and Levy (8) found that the buckling load is increased with normal pressure, but not as much as in the case of the simply supported plate. Specific results for three pressures which they investigated are as follows:

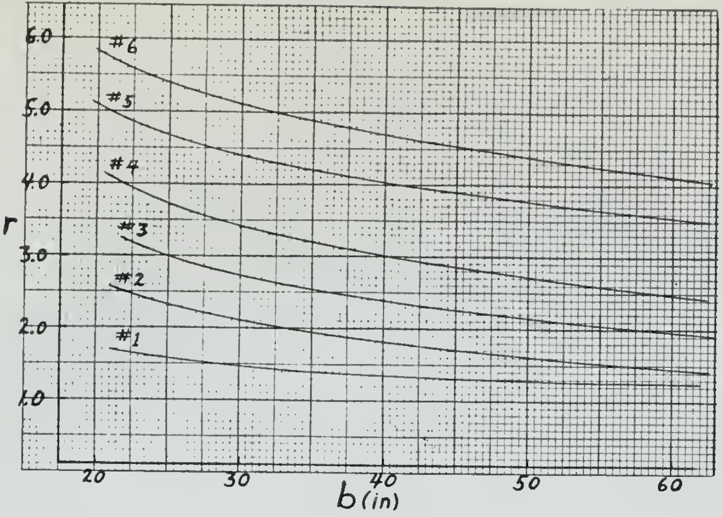

FIGURE 11 - Radius of Gyration of 1/2" Plate and Selected Stiffener Combinations vs. Stiffener Spacing.

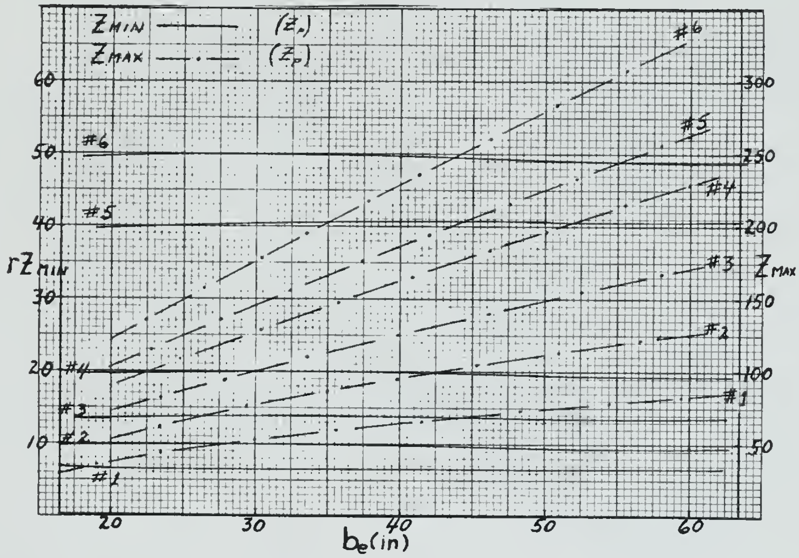

FIGURE 12 - Section Moduli of 1/2" Plate and Selected Stiffener Combinations versus Effective Breadth.

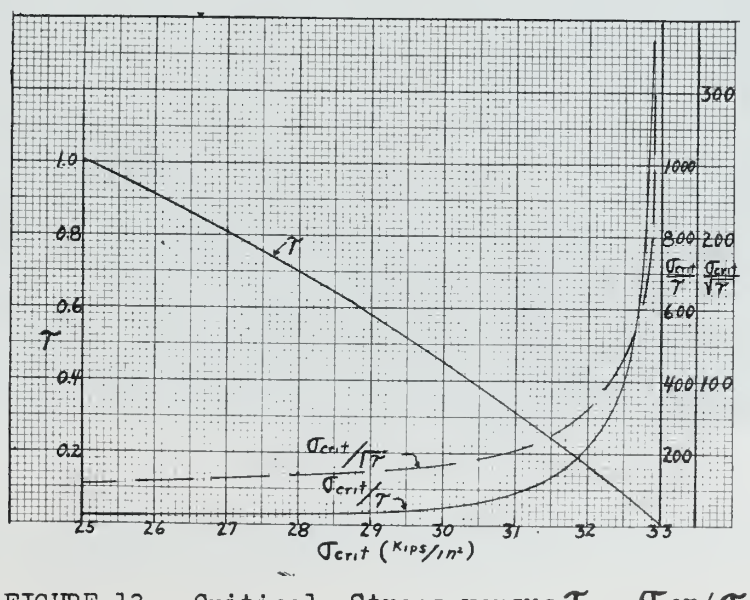

FIGURE 13 - Critical Stress versus τ , $\sigma cr/\tau$ and $\sigma cr/\sqrt{\tau}$ for Typical Mild Steel

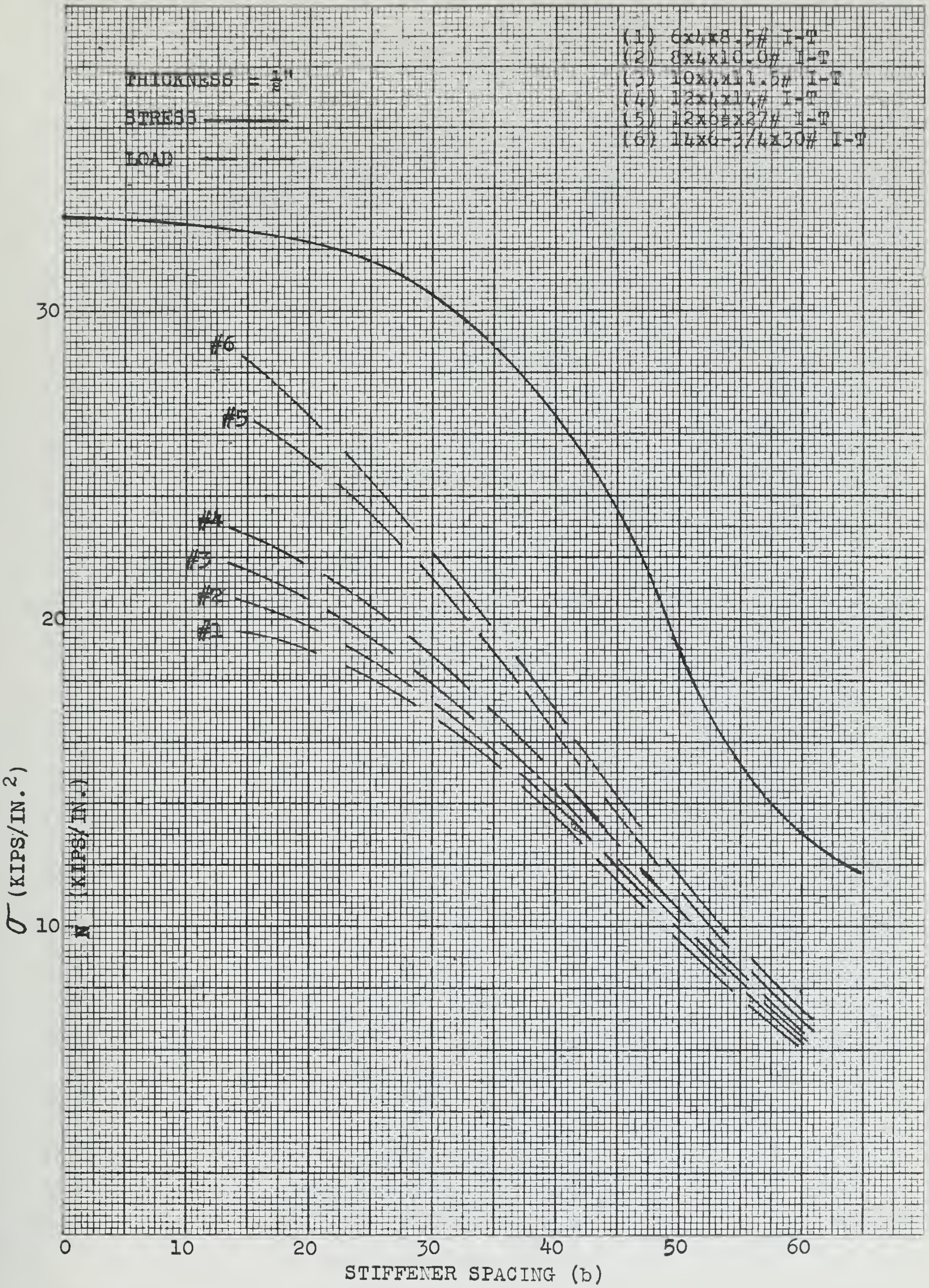

FIGURE 14 - Design Stress and Design Load versus Stiffener Spacing for 1/2" Plate and Selected Stiffener Combinations.

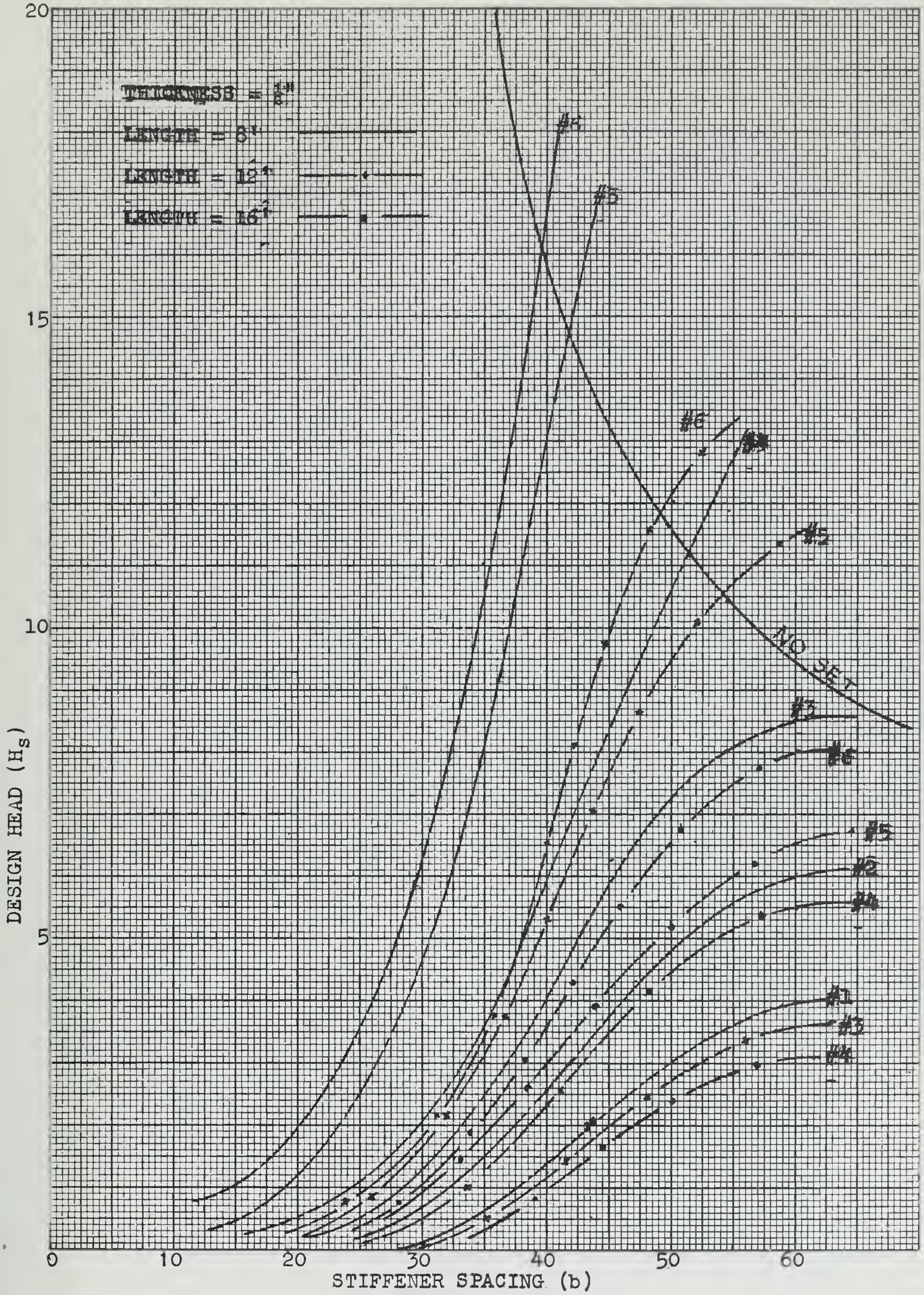

FIGURE 15 - Design Head (Acting on Stiffener Side of Plate) versus Stiffener Spacing for 1/2" Plate and Selected Stiffener Combinations. Note: No Set Curve Indicates Yield Stress in Plate Due to Lateral Load.

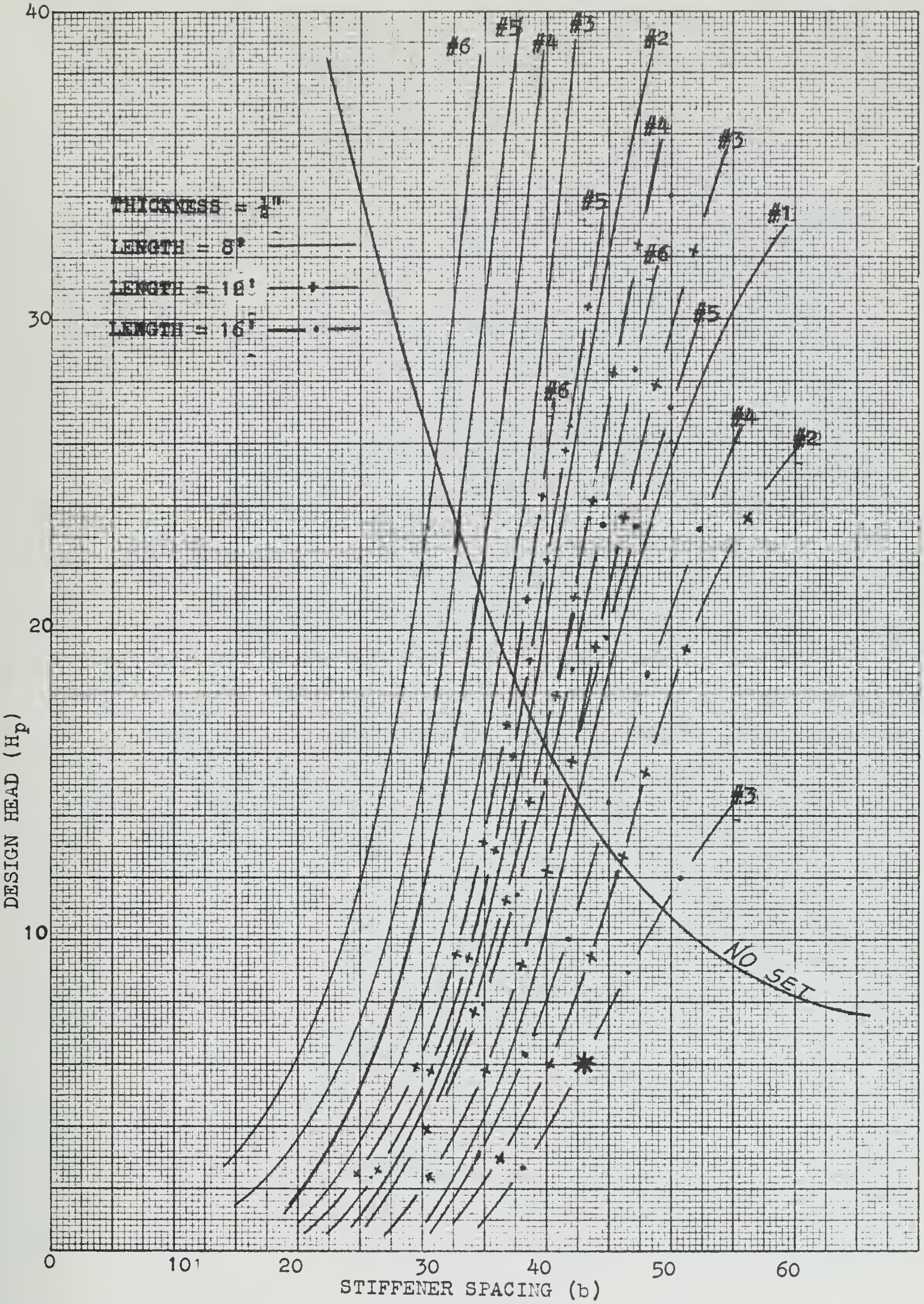

FIGURE 16 - Design Head (Acting on Smooth Side of Plate) versus Stiffener Spacing for 1/2" Plate and Selected Stiffener Combinations. Notes: (1) No Set Curve Indicates Yield Stress in Plate Due to Lateral Load (2)* Indicates That Bending Stress in Stiffener Flange Exceeds Yield Stress Beyond This Point.

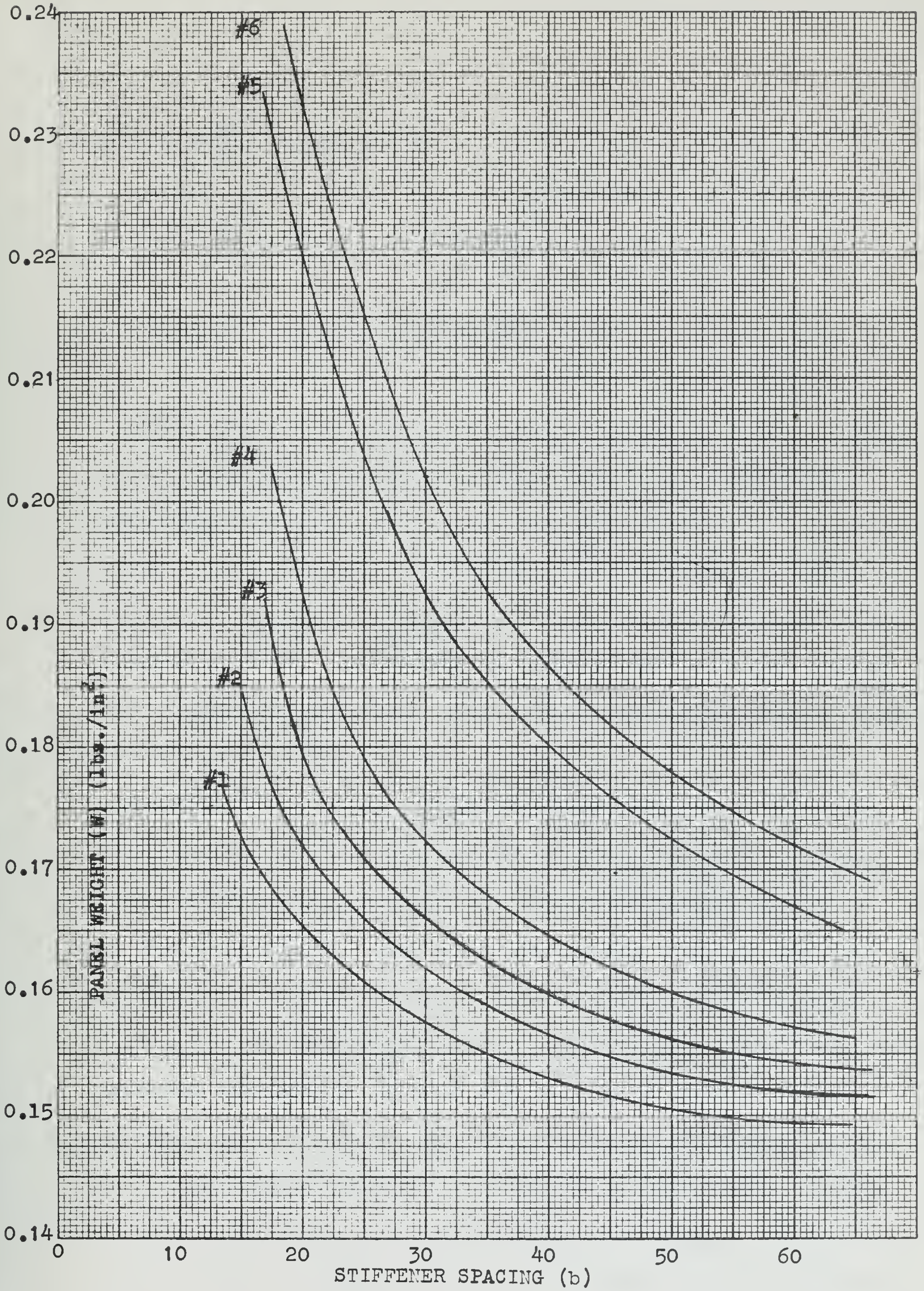

FIGURE 17 - Weight of Panel per Unit Length and per Unit Breadth versus Stiffener Spacing for 1/2" Plate and Selected Stiffener Combinations.

$$P = 6.4 \frac{Et^3}{b} \quad \text{when } p = 0$$

$$P = 6.8 \frac{Et^3}{b} \quad \text{when } p = 15.02 \frac{Et^4}{b^4}$$

$$P = 8.3 \frac{Et^3}{b} \quad \text{when } p = 37.55 \frac{Et^4}{b^4}$$

Thus, for the highest pressure considered on the clamped plate, the theoretical buckling load is 1.3 times the buckling load for zero normal pressure.

For a comparison of the above results Figure 4 shows a plot of $\frac{P}{Et^3/b}$ vs. $\frac{p}{Et^4/b^4}$ for both simply supported and clamped plates. According to this figure clamped plates have a smaller buckling stress for pressures greater than about 7.4 Et^4/b^4.

Because the plating under consideration is continuous over equally spaced stiffeners, and is subject to normal load, clamped edge conditions exist at edges contiguous to the stiffeners. The question arises whether the increase in buckling strength due to normal load is significant for clamped conditions. Bleich (9) covers this question adequately in his discussion of ships' bottom plating; he shows that the deflection to thickness ratio, f/t, for clamped plates is always less than 0.4. By use of this limitation and by plotting the results of Figure 4 on the basis of $\frac{\sigma_{cr}}{\sigma_{cr\,0}}$ vs. f/t, he shows that the increase

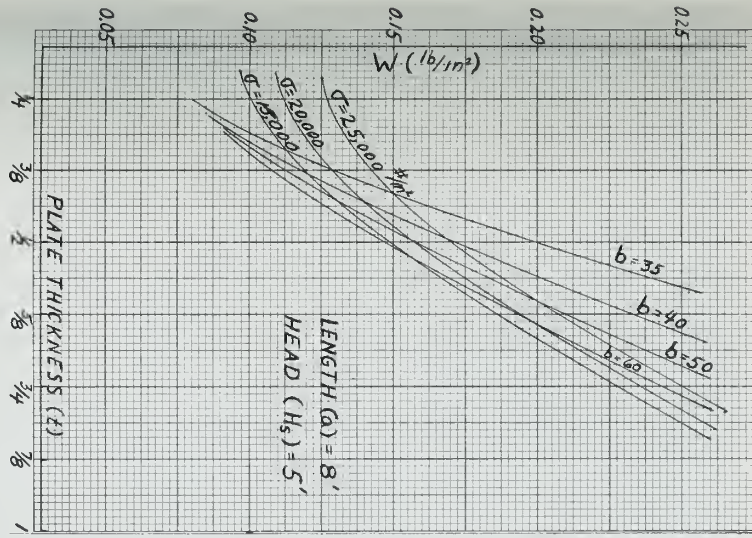

FIGURE 18 - Weight of Panel per Unit Length and per Unit Breadth vs. Plate Thickness for Specified Design Stress, Design Head, Panel Length and Stiffener Spacing.

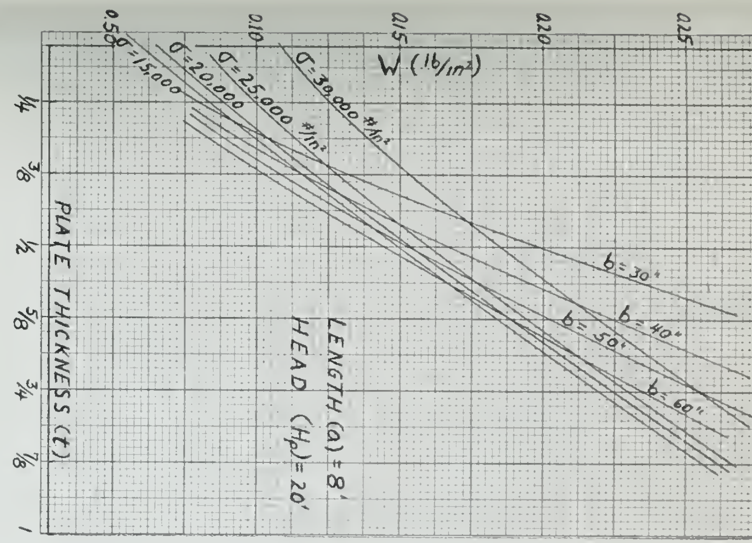

FIGURE 19 - Weight of Panel per Unit Length and per Unit Breadth vs. Plate Thickness for Specified Design Stress, Design Head, Panel Length and Stiffener Spacing.

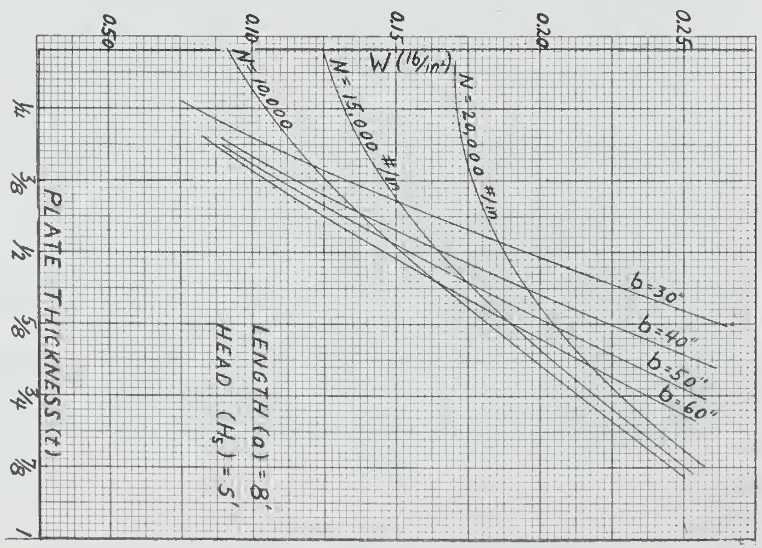

FIGURE 20 - Weight of Panel per Unit Length and per Unit Breadth vs. Plate Thickness for Specified Design Load, Design Head, Panel Length and Stiffener Spacing.

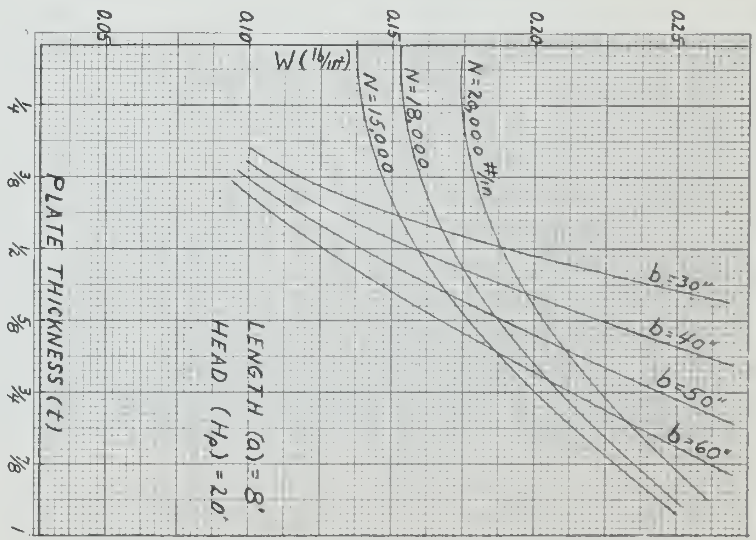

FIGURE 21 - Weight of Panel per Unit Length and per unit Breadth vs. Plate Thickness for Specified Design Load, Design Head, Panel Length and Stiffener Spacing.

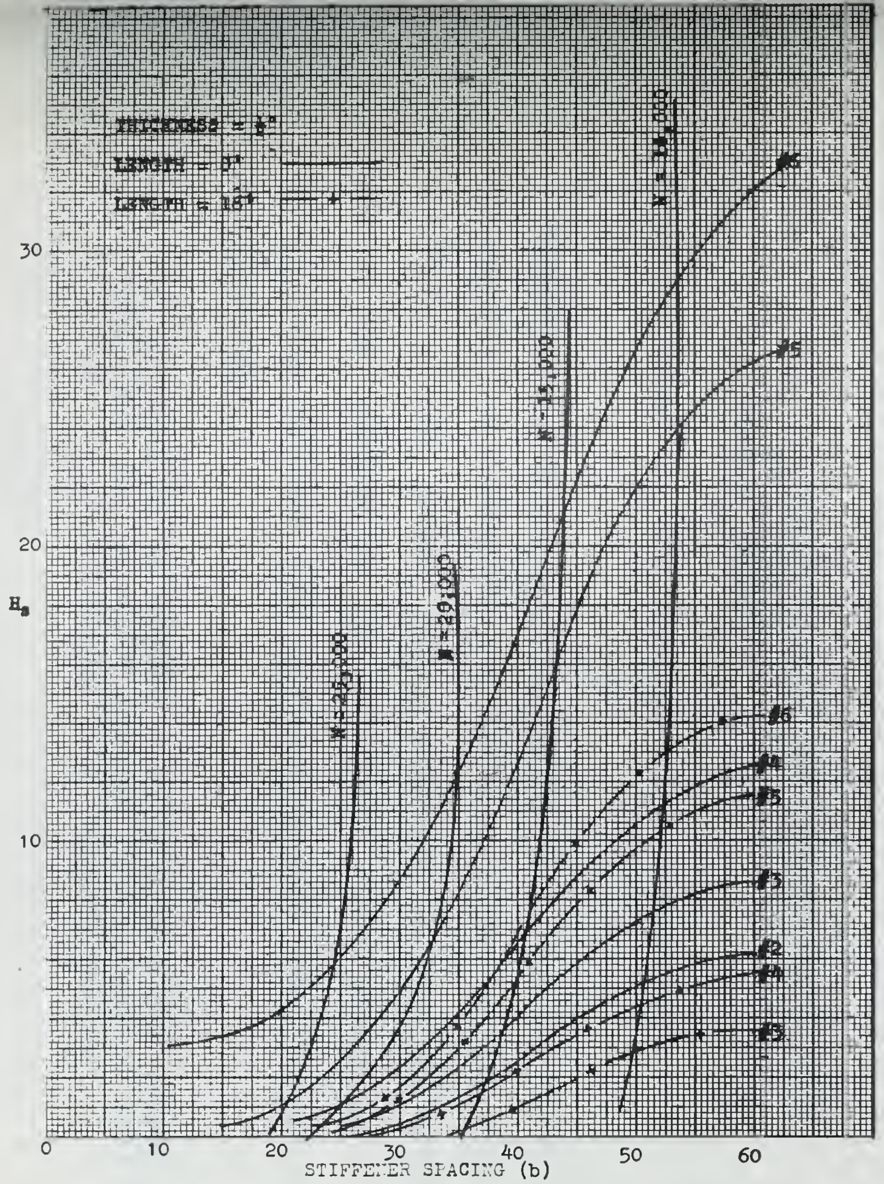

FIGURE 22 - Design Head (Acting on Stiffener Side of Panel) versus Stiffener Spacing for 1/2" Plate and Selected Stiffener Combinations with Specified Design Loads and Panel Lengths

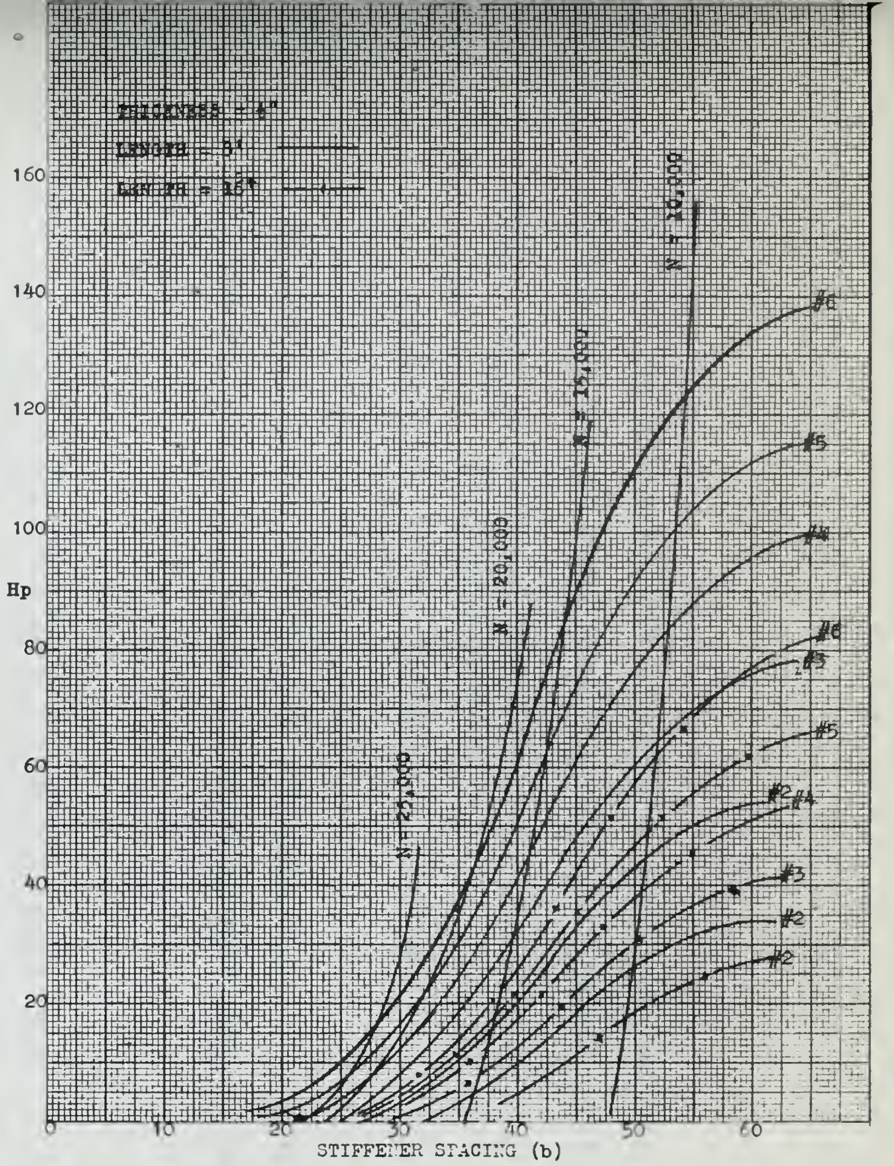

FIGURE 23 - Design Head (Acting on Smooth Side of Plate) versus Stiffener Spacing for 1/2" Plate and Selected Stiffener Combinations with Specified Design Loads and Panel Lengths

in buckling strength due to normal load for clamped conditions may be neglected.[1] Figure 5 shows this plot.

where: σ_{cr} = buckling stress under the action of normal pressure.

σ_{cr0} = buckling stress for 0 normal pressure.

By making these assumptions the appropriate relation to express the local plate critical stress can be written as:

$$\sigma_{cr\ p} = \frac{\pi^2 E \sqrt{\tau_p}}{12(1-\mu^2)} \frac{K_p \ t^2}{b^2} \qquad \text{(ref. 9, page 322)} \qquad (4)$$

where:

K_p = 6.97 for clamped edge conditions contiguous to the stiffeners and simple supports at the loaded edges.

Figure 6 gives K_p for various edge conditions.

Equating Element Buckling Strengths. Having selected appropriate expressions for the stability of the component elements of the panel, it is possible to combine them by use of the optimum weight criterion--that buckling of the local plate element and stiffener-plate combination occur simultaneously.

---

1. The conversion of Figure 4 to Figure 5 for clamped conditions may be made through Timoshenko's (10) relation for deflection of a clamped plate:
$f = .0277 \frac{pb^4}{Et^3}$.

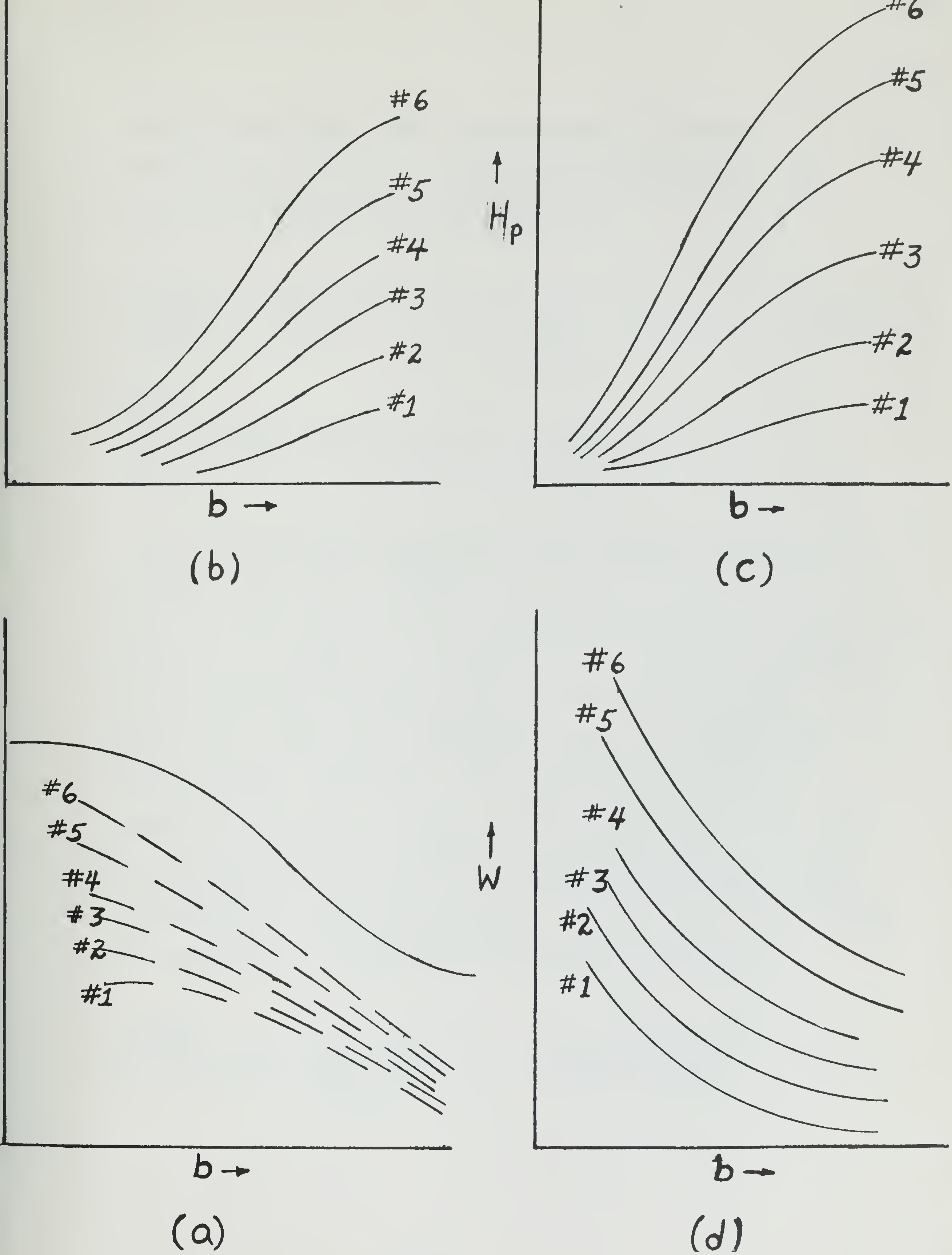

FIGURE 24 - Proposed Design Chart Arrangement

Before combining these expressions it is helpful to consider how a factor of safety can be included. The interaction formula can be written as:

$$\frac{F.S._c \, \sigma_a}{\sigma_{cr\ col}} + \frac{\sigma_b \, F.S._b}{\sigma_{b\ all}} = 1; \text{ or } F.S._c \, \sigma_a = \sigma_{cr\ col}\left[1 - \frac{F.S._b \, \sigma_b}{\sigma_{b\ all}}\right] \qquad (6)$$

where:

$F.S._c$ = factor of safety in compression.

$F.S._b$ = factor of safety in bending.

This implies that buckling of the stiffener-plate combination (as a beam-column) occurs when an axial stress equal to $F.S._c(\sigma_a)$ is reached in the stiffener, assuming it is simultaneously subjected to the bending stress $F.S._b(\sigma_b)$. From the statement of the problem and the optimum weight condition, the stress in the plate is equal to $F.S._c(\sigma_a) = \sigma_{cr\ p}$. The axial stress $F.S._c(\sigma_a)$ can appropriately be called the <u>compressive design stress</u>.

Substituting the expression for the plate critical stress into Equation (6) yields the following expression:

$$\frac{\pi^2 E\sqrt{\tau_p}}{12(1-\mu^2)} K_p \left(\frac{t}{b}\right)^2 = \frac{\pi^2 E \ \tau_{col}}{\left(\frac{K_c \ a}{r}\right)^2}\left[1 - \frac{\gamma H \ b \ a^2 \ F.S._b}{K_b (144) Z \, \sigma_y}\right] \qquad (7)$$

Choosing the Independent Variables. A way of manipulating the above equation into some useful form is not readily apparent. It must be realized that the τ values associated with plate and stiffener are not the same. This at first may seem contradictory since it has been previously specified that the stress is the same in the stiffener and in the plate. The explanation lies in the fact that $\sigma_{cr\ col}$ is a fictitious stress which is in fact never reached; and the value of τ_{col} is of course a function of this stress, which in turn is a function of the stiffener a/r only as shown in Figure 2.

After many attempts at trying to find a useful form of Equation (7), it was found that the most useful independent variables to select were plate thickness, stiffener spacing, stiffener geometry, and length of panel. (Material must of course be selected also.) The remaining variable, Head, can be solved in terms of the other variables and a first degree equation in Head results. Solving the equation for Head results in the following:

$$(F.S._b)H = \frac{(144)\sigma_y Z K_b}{\gamma ba^2} \left[1 - \frac{\dfrac{\pi^2 E \sqrt{\tau_p}\, K_p \left(\frac{t}{b}\right)^2}{12(1-\mu^2)}}{\dfrac{\pi^2 E\, \tau_{col}}{\left(\dfrac{K_c\, a}{r}\right)^2}} \right] \qquad (8)$$

where:

$F.S._b(H) = H_{des}$

Cancellations have purposely been avoided so that the equation may be expressed in the compact form:

$$(F.S._b)H = \frac{(144)\sigma_y \ \ z \ K_b}{\gamma ba^2} \left[1 - \frac{\sigma_{cr\ p}}{\sigma_{cr\ col}} \right] \qquad (9)$$

where:

$\sigma_{cr\ p} = F.S._c(\sigma_a) =$ design stress

It is helpful to keep in mind the factors on which each of the major terms are dependent for a given material, end conditions, and fluid.

(a) Design stress is a function of plate thickness and spacing alone. (t and b).

(b) $\sigma_{cr\ col}$ is a function of the stiffener geometry, unsupported length, and plate thickness. (Plate thickness has an effect because it is the r of the stiffener-plate combination which is of concern.)

(c) The factor multiplying the brackets is a function of the stiffener geometry, stiffener spacing, length, and plate thickness.

Since the selection of the independent variables specifies the weight of the panel and plate critical or design stress, it is possible through judicious selection of a limited number of independent variables to obtain useful plots of the weight-strength relationships as functions of the major parameters--particularly plate thickness and stiffener spacing.

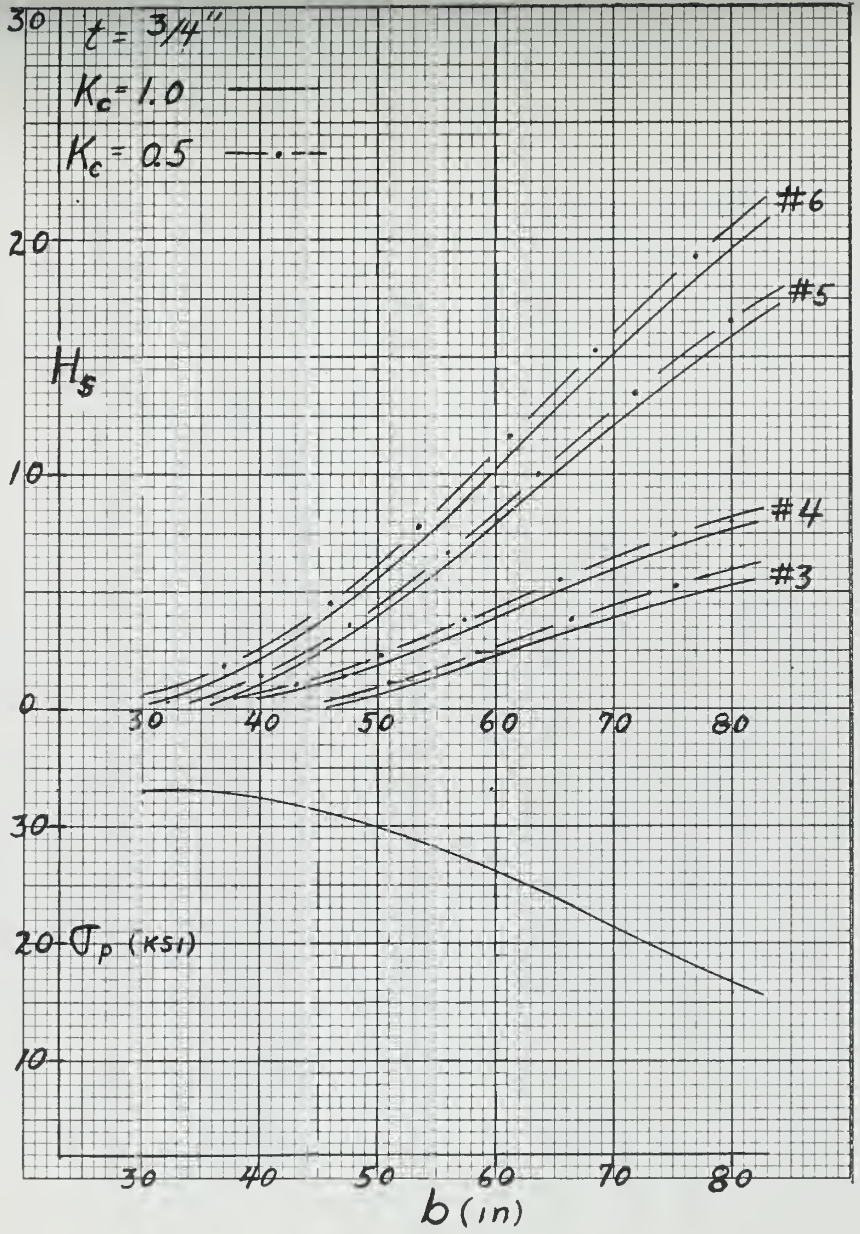

FIGURE 25 - Effect of Stiffener End Constraint Coefficient (K_c).

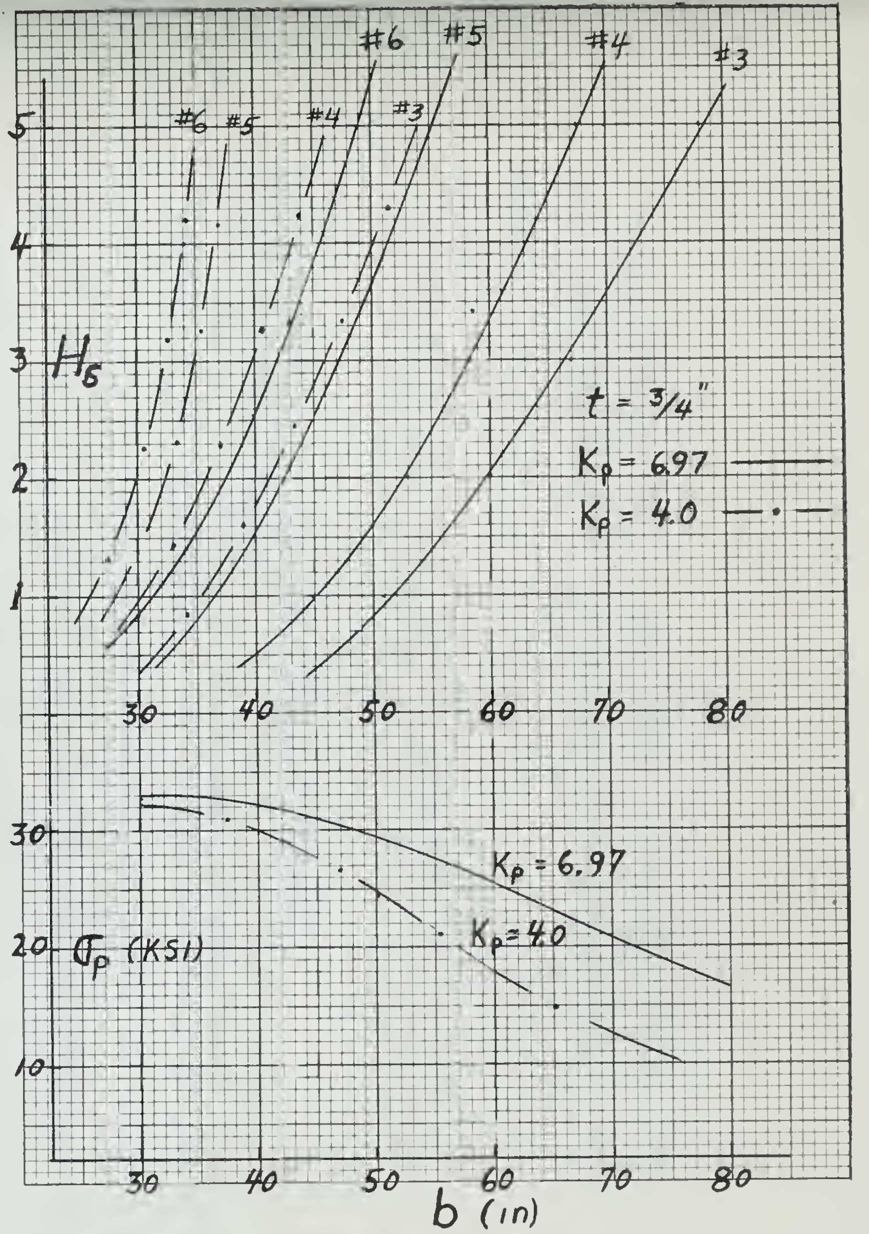

FIGURE 26 - Effect of Plating Edge Constraint Coefficient (K_p).

Panel Weight. It is convenient to express the weight of the panel in terms of weight per unit breadth per unit of panel.

$$W = \gamma(bt + A_s)\frac{1}{b} \qquad (\text{lbs./in.}^2)$$

Application to a Combination of Standard Plates and Stiffeners

Selection of Stiffeners. It is now possible to apply Equation (8) to some typical panels in order to determine typical weight-strength relationships for various geometries.

Although some ideal series of stiffeners could be assumed, it was decided that the analysis would be more practical if a real series of stiffeners were selected. A stiffener type commonly used in naval construction has been selected--specifically, T-type stiffeners cut from standard I-beams.

Since this is an attempt to optimize weight, it was considered necessary to find a set of stiffeners, covering a practical range, which appeared to offer the best properties for a given weight. Inspection of Equation (8) shows that stiffeners which give a high r and Z when combined with their effective plating are to be desired. Therefore plots of Z and r vs. stiffener area were made for all available T stiffeners in conjunction with various plate thicknesses and effective amounts of plating; Figures 7,

8, and 9 are examples of such plots. The stiffeners which are circled and numbered on these plots were selected for use in this analysis because they satisfied two basic criteria.

a.) They appeared to give the best structural efficiency, i.e., the highest Z and r for a given cross sectional area.

b.) They were rather evenly spaced throughout a broad range--a characteristic desirable for covering a spectrum of loadings and geometry.

The fact that these stiffeners consistently gave the highest Z's and r's for a given amount of material over a wide range of plate thicknesses and effective widths (or breadths) emphasizes that solutions obtained using these stiffeners represent the optimum weight obtainable with T-type stiffeners.

Specifically, the sections selected were:

(1)	6x4x8.5 lb. I-T	(5)	12x6½x27 lb. I-T
(2)	8x4x10.0 lb. I-T	(6)	14x6-3/4x30 lb. I-T
(3)	10x4x11.5 lb. I-T	(7)	16x7x37 lb. I-T
(4)	12x4x14 lb. I-T	(8)	18x7½x50 lb. I-T

Figure 10 is a sketch of an I-T stiffener.

Solution of the Head Equation for the Real Stiffeners. The Head Equation (8) can be solved for each of the eight selected stiffeners for various combinations of length, plate thickness, material, and stiffener spacing. It was decided to work only with mild steel in this application thus reducing the number of variables to a more reasonable number.

Because properties of the plate-stiffener combinations could be found tabulated for the standard plate thicknesses at several values of effective width and breadth, it was simplest to fix plate thickness and length, then solve for Head using several values of spacing. Properties of the plate-stiffener combinations were available for effective widths and breadths of 29t, 51t, and 60t. It was found that these were insufficient to cover the range desired, so that several additional values were computed and curves of the properties were drawn up for each of the plate thicknesses considered. (The properties of the selected stiffeners combined with 1/4", 1/2", and 3/4" plate are summarized in Appendix C, table C-III.) Having these curves it was a simple matter to enter with either effective width or breadth and read off the appropriate property of the section. Typical examples of these curves are shown in Figures 11 and 12. It is necessary to obtain the effective breadth (a function

of a and b--Figure 3) before reading off the section modulus of the section. Having the modulus and radius of gyration, it is a simple matter to enter the plate critical stress curve and column curve to obtain the necessary values to solve the Head Equation. Values of the weight, W, and Load, N, are worked out simultaneously for the same range of b and t. Appendix B gives a step-by-step procedure for solving the Head Equation.

The results obtained from solving the Head Equation are plotted and curves such as shown in Figures 14, 15, 16, and 17 are the result.

Checking the Limits of Validity of the Failure Theory. The solution at this point does not preclude failure of the plate due to excessive tension from the action of the lateral load. Nor does it preclude failure of the stiffener due to excessive tension in the flange when the lateral load is on the side of the plate. Since the analysis assumed the failure was of an instability type, both of these other types of failure may create a limit of validity for the failure analysis. For example, the limits of validity may govern in the case of a bulkhead in which the axial compressive load is relatively small and the lateral bending load is very large, and where it is acceptable to design on the basis of allowing rather large bending deformations to occur. The establishment of these validity limits is

a simple matter, but requires a design decision as to what constitutes a tension failure. Limits have been chosen in this report based on the stiffener flange reaching yield stress and the plating reaching yield stress in tension. Figure 16 shows the nature of these limits.

III. RESULTS

The specific combinations investigated by the authors are as follows:

Material .... mild steel

Yield Point .... 33,000 p.s.i.

Proportional Limit .... 25,000 p.s.i.

τ and $\sqrt{\tau}$ as shown in Figure 13.

Stiffeners as listed on page 12.

Fluid causing lateral pressure load--Salt Water (γ =64 lbs./ft.^{3})

Loaded edge conditions: Simple Supports

t = 1/4"	t = 1/2"	t = 3/4"
a = 8, 12 ft.	a = 8,12,16 ft.	a = 8,12,16 ft.
b = 10" thru 60"	b = 20" thru 60"	b = 20" thru 80"

Because the purpose of this report is primarily to show a method, only selected results have been chosen for representation in graphical form--namely, the results of 1/2 inch plate; they are adequate to show the method and the general nature of the resulting curves. A summary of the results for all combinations investigated is included in tabular form in Appendix C, Table C-II. The remainder of this re-

port uses the graphical results of 1/2 inch plate to show the nature and use of the strength-weight relations obtained by using the foregoing analysis and method of solution.

The results obtained from this type of analysis are presented in the form of four plots which show the effect of all the variables; these plots are:

1. Design stress or plate critical stress vs. stiffener spacing, and nominal Load, N, vs. Stiffener spacing. (Figure 14.)

2. H_f vs. stiffener spacing for lateral load on the same side of the panel as the stiffeners. (Figure 15.)

3. H_p vs. stiffener spacing for lateral load on the plate side of the panel. (Figure 16.)

4. Weight, W, (lbs. per unit breadth per unit length) vs. stiffener spacing. (Figure 17.)

IV. DISCUSSION OF RESULTS

All of the above curves are plotted for a given thickness of plating, length of panel, and specific selected stiffeners, except that weight and stress are not a function of length, nor is stress a function of the stiffeners used.

The first of these, Figure 14, shows that for a given plate thickness, specifying the design stress implies that there is only one frame spacing which will satisfy the optimum weight criterion; this spacing can be read directly from the plate critical stress curve. On the other hand, specifying the axial Load, N, does not fix the spacing because there are several different combinations of stiffeners and spacing which correspond to the axial Load. Likewise, specifying the spacing fixes the design stress. As would be expected, wide spacings correspond to low values of design stress.

The second and third curves, Figures 15 and 16, show that a value of stiffener spacing together with Design Head selects a stiffener to satisfy the optimum weight criterion--the design stress for any combination of H and b obviously is fixed for each different value of b.

It may seem paradoxical that these curves show that a stiffener can withstand greater heads for wider spacing, but it must be remembered that these wider spacings automatically imply lower axial design stresses since each point on the curves satisfies the optimum weight criterion.

The curve of weight versus stiffener spacing shows that for any given stiffener the weight is less for wide spacings, but again it must be remembered that these wide spacings correspond to low values of the design stress; therefore, the ratio of axial stress to weight also becomes low.

Figures 18 and 19 are plots of weight vs. plate thickness for specified values of design stress, Head, and panel length. Figures 20 and 21 are similar except they are for specified values of the Load, N. These curves are obtained by cross-plotting the information from the curves shown for 1/2" plate together with information from similar curves for various plate thicknesses. <u>Curves such as these perhaps best show the overall trends for optimum stiffener spacing and size</u>. For example, they show graphically that the minimum weight for a given stress, Head, and length occurs in all cases for the thinnest plates together with small stiffener spacings. Another important feature of these curves of weight vs.

thickness is that when a minimum spacing is specified for design reasons, the curves indicate the optimum plate thickness and the minimum weight attainable for the design. This feature is useful when the minimum spacing may be limited by construction costs, access, etc.

In the actual design process this minimum weight may not be attainable if the optimum weight solution specifies a point not on an available real stiffener; but even so, this minimum weight could be used as a measure of merit for the design.

Figures 22 and 23 are simply Figures 15 and 16 replotted with lines of constant axial Load, N, derived from Figure 14. This form of presentation is useful when design is being done knowing axial Load and Head. The results plotted in this manner indicate that the intersection of the desired Head and Load, as in the case of stress and Head, indicate directly the required stiffener spacing and size to satisfy the optimum weight criterion.

Use of Curves in Design

The proposed arrangement of the results for use as design charts is as shown in Figure 24. This is simply an arrangement of Figures 14 through 17 in a manner suitable for direct reading in a practical design problem. One such arrangement would be required for each plate thickness;

and the curves of Head vs. stiffener spacing would be required for various unsupported lengths in the range of interest. For example, three lengths were included in Figures 15 and 16. In this manner 12 of these design charts can cover completely the standard thickness sizes for 1/4 inch through 1.0 inch plating for three or four unsupported lengths. The number of unsupported lengths which must be included will be governed by practical consideration of the structural type for which the charts will be used.

In discussing the design problem it is assumed that the designer will know the required axial stress or Load, N, the lateral Head, H, and the unsupported length. When designing on the basis of a given design stress, the curves can be entered as follows:

1.) Enter Figure 24-a with the given stress; from the curve read off the corresponding value of b.

2.) Using the value of b obtained above, enter Figure 24-b or 24-c depending on which side of the panel is subject to lateral load; the intersection of this spacing with the given design Head establishes the size of stiffener satisfying the minimum weight criterion.

3.) Enter Figure 24-d with the spacing and size of stiffener as obtained above and read off the value of weight.

4.) Repeat the above steps for various plate thicknesses and compare the values of weight and spacing obtained.

At first glance this may seem like a rather long sorting process. In the practical design problem, however, the limitation of a minimum acceptable stiffener spacing would immediately eliminate many of the smaller plate thicknesses. This limitation, in conjunction with the fact that the least weight solution requires the smallest possible plate thickness, would allow rapid selection of the optimum plate thickness, stiffener size, and spacing.

When designing on the basis of a given applied Load, N, instead of a design stress, it is possible to use this same arrangement of curves; the curves are entered as follows:

1.) Figure 24-a is entered with the design Load; simultaneously Figure 24-b or 24-c (depending upon the application of the lateral load) is entered with the value of design Head.

2.) The correct solution to the optimum weight criterion for the given loadings is obtained by finding the stiffener which intersects both the head and load at the same value of spacing. A few trials may be necessary to find this value of spacing.

3.) Again, the weight of the stiffener-plate combination is read off of Figure 24-c by entering with the value of stiffener size and spacing as obtained above.

As was pointed out earlier, Figures such as 22 may be more easily used when designing on the basis of Load since it becomes unnecessary to use a trial and error procedure to establish the correct value of stiffener spacing.

In the actual design problem the design Head and required spacing may not intersect at one of the selected stiffeners used in the solution. Obviously when this happens, the method will not give an exact solution to the design problem. An approximation may be obtained, however, by using the intersection between real stiffeners to select the spacing; and by using this spacing an approximate weight can be obtained from Figure 24-d. The weight estimated in this manner will of course be the minimum weight attainable for the particular plate thickness under consideration because the stiffeners chosen for this solution were the best on a weight basis. This optimum weight can be used as a basis for the selection of the best plate thickness and stiffener spacing for the problem. Knowing the properties of stiffeners selected for the solution, the designer may also estimate the desired properties for the stiffener needed in his actual design. For example, if the design stress and Head call for a stiffener midway between No. 1 and No. 2, the desired properties (I, Z, r, etc.)

will certainly lie between the properties of No. 1 and No. 2 for that spacing.

With this information available the designer can more easily choose the best available stiffener using the design chart weight as a basis of comparison. It may happen that when another stiffener is selected, the resulting panel weight will exceed the weight corresponding to the next larger stiffener on the design chart. When this happens, it is possible to achieve a higher design head, or stress, or both with no increase and possibly a decrease in the panel weight. In other words, for certain stress-Head combinations a greater factor of safety may be obtained with no increase in the panel weight merely by the use of a more efficient stiffener. Although this type of design chart will not give the user the exact answer in a large number of the practical design problems, its use will aid him greatly in the choice of the correct plate thickness, stiffener size, and spacing for an optimum weight solution.

It would be helpful to compare the results obtained in this investigation with experimentally determined results, but there are none in the range of interest and therefore no conclusive evaluation of the results is possible on this basis. A testing program is in progress, however, (at Lehigh University) to evaluate

the behavior of longitudinally stiffened plates subjected to axial and normal loads. When the results of this testing program become available, it is recommended that the experimental results be compared with the analysis as presented in this report. Caution should be taken, however, when comparing experimental results with this analysis since it assumes general instability, i.e., buckling of the beam-column and plate simultaneously.

Although no conclusive evaluation of the results of this analysis with experimental results is possible, it is felt that the concepts used in the development of this design method are generally accepted in engineering design, and the results obtainable are accurate within the normal limits of engineering requirements.

Based on the foregoing discussion the authors feel it is appropriate to recommend highly the subject method for use in the design of actual ship structures.

V. CONCLUSIONS

1. The use of the optimum weight condition allows simple structural design equations to be combined into a simple design formula.

2. Use of standard plates and stiffeners permits a practical solution of the design formula and yields results which can be presented in the form of direct reading design charts that allow selection of stiffener size and spacing and plate thickness for optimum weight given: axial compressive stress (or load), head, and length of panel.

3. Through simple cross plotting of the basic results of the design formula additional curves may be derived which show the overall trend of weight versus such design parameters as plate thickness and stiffener spacing. These plots should be valuable in preliminary design as a rational basis for the selection of these variables.

4. Because of the assumption of uniform compressive stress in the condition of loading, and because the boundary con-

ditions and geometry assumed in the analysis closely approximate the conditions in decks and bottom plating of longitudinally framed ships, this method should be particularly useful in the design of these elements.

5. The accuracy of the results of this method should be within the normal limits of engineering requirements because the component elements of the analysis are all well-established engineering formulations and approximations.

VI. RECOMMENDATIONS

1. When experimental results become available this analysis should be compared with experiment to determine the degree of agreement.

2. Results for a more detailed coverage of the range of interest should be calculated so that the results can be used in design. Specifically, a complete coverage of all the standard size plate thicknesses are needed in order to make it possible to use the method for design. A more detailed coverage of cross-plots such as Figures 18, 19, 20, and 21 would be a convenient adjunct to design.

3. It is recommended that the subject method actually be used in design of appropriate ship structures--specifically deck and bottom plating of longitudinally framed ships.

VII. APPENDICES

Appendix A. Effect of Boundary Condition Constants

As mentioned previously there are several constants assumed in the solution of this problem. Three of these constants are dictated by the boundary conditions or edge constraints of the plating and stiffener--K_b, K_c, and K_p. The values of these constants and the Head equation (Equation 8) are listed again below to facilitate reference during the following discussion:

$$H_{des} = (F.S._b)H = \frac{\sigma_y K_b Z(144)}{\gamma ba^2} \left[1 - \frac{\dfrac{\pi^2 E \sqrt{\tau_p}\, K_p \left(\frac{t}{b}\right)^2}{12(1-\mu^2)}}{\dfrac{\pi^2 E\, \tau_{col}}{\left(\dfrac{K_c a}{r}\right)^2}} \right]$$

Constant	Condition assumed in analysis	Corresponding value
K_b	simply supported	8.0
K_c	simply supported	1.0
K_p	local plate panel clamped at unloaded edges--simply supported at loaded edges.	6.97

The boundary condition constants--K_b, K_c, and K_p--assumed in this analysis define a specific set of boundary conditions; the general usefulness of the results are therefore limited. Since the problem may often call for edge constraints different from those assumed in this analysis, it is useful to see how changes in these constants affect the solution.

Equation (8) indicates that the design Head is directly proportional to the constant K_b. Therefore, if the designer wishes to assume end conditions for the stiffener other than simply supported (pinned ends), he merely has to multiply the desired design Head by the ratio of $\frac{8}{K_b}$. For example, suppose that the stiffeners were to be firmly bracketed at the ends to heavy members such that their action would be more nearly clamped than simply supported; the design K_b would then be 12 instead of 8. To carry the example a step further, if the desired design Head were 10 feet, the design charts would be entered with a Head of 10x8/12=6.7 feet. In considering this example it should be noted that it would not be geometrically consistent to use a K_b corresponding to clamped conditions without using a constraint coefficient for column buckling (K_c) also corresponding to clamped conditions.

This naturally leads to a consideration of the effect of K_c. Equation (8) shows that design Head does not vary

directly with K_c; there is therefore no simple way of considering changes of K_c on the design charts. In ship structures the range of K_c of most interest is from 1.0 to 0.5--corresponding to pinned ends and clamped ends respectively. In order to evaluate the effect of K_c a solution was worked out for $K_c = 0.5$ holding all other constants the same as in the original solution. The results of this calculation for one plate thickness are presented in Figure 25. The design Head for a given stiffener is only slightly greater for $K_c = 0.5$ than for $K_c = 1.0$. This increase in design Head is in the order of only $\frac{0.5}{10}$ or 5% of the Head for the case of pinned ends. The design Head for the pinned end column ($K_c = 1.0$) is therefore a good although slightly conservative estimate of the clamped-end condition, and it is suggested that for a first approximation--especially in the inelastic range--the small effect of a change in K_c may be neglected and the design charts used without change.

The effect of a change in the plate edge constraint coefficient, K_p, remains to be investigated. Equation (8) again shows that the design Head does not vary directly with K_p. In order to show the effect of changing K_p, the solution was carried out for $K_p = 4.0$ (all plate edges simply supported. The results are shown in Figure 26.

This solution has only been carried out for small Heads because it is felt that only for small Heads could the edge action actually approximate simply supported conditions. For these small Heads the increase in the critical buckling stress of the plate due to the lateral load is small enough to be neglected, especially in this discussion which is only intended to indicate the general effect of a change in K_p.

Figure 26 indicates that changing the plate constraint constant, K_p, defines an entirely different problem than the one assumed in this analysis, and therefore a separate solution would be required to take account of the change.

An interesting consideration which the designer must bear in mind when using this analysis for design is illustrated by Figure 26--that in case the lateral load is removed, the plating may react as though it were simply supported contiguous to stiffeners with a consequent reduction in the critical buckling stress. When this type of service condition is expected, the factor of safety on compression should be sufficient to ensure that the actual stress to which the plate is subjected will be less than the critical buckling stress for simply supported edge conditions.

Appendix B. Steps in Solution of Head Equation (8)

1. Select appropriate stiffeners to use.

2. Select plate thickness and length.

3. Plot properties of stiffeners vs. effective width (or effective breadth) of plating. (Z_f, Z_p, and r vs. b and b_e.)

4. Select the 1st stiffener for which the solution is to be made.

5. Pick a value of b.

6. Enter Vedeler's curve of effective breadth to obtain effective breadth in bending.

7. Enter curve of r vs. b and pick off a value of r.

8. Using value of b selected above, enter curve of plate critical stress vs. b, and pick off a value of plate critical stress.

9. Compute Ka/r and enter curve of column critical stress vs. Ka/r and read off value of column critical stress.

10. Substitute the necessary values into Equation (8) and solve for Head.

11. Compute a value of Load corresponding to the above plate critical stress and geometry of structure.

12. Compute the weight of the above combination.

13. Check the value of tensile stress in the flange when load is on the plate side.

14. Repeat steps 4 through 13 for various values of b in the range of interest.

15. Repeat all of the above steps for various thicknesses and lengths of interest.

16. Repeat all of the above for various thicknesses and lengths of interest.

17. Plot the results.

18. Compute the limit of validity of buckling failure corresponding to excessive tensile stresses in the plate. This can be done by plotting points on the H vs. b curve for combinations of H and b that give yield stress in the plate.

APPENDIX C Summary of Data and Calculations

Table C-I	Plate critical stress as calculated by equation (4)
Table C-II	Summary of design heads (H_s and H_p) for 1/4", 1/2" and 3/4" plate and selected stiffeners as calculated by equation (8).
Table C-III	Properties of selected stiffeners plus an effective amount of plating (s) for 1/4", 1/2" and 3/4" plate.

SUMMARY OF CALCULATIONS

TABLE C-I

t	1/4"	1/2"	3/4"
b	$\sigma_{cr\ p}$	$\sigma_{cr\ p}$	$\sigma_{cr\ p}$
10	32,500	--	--
15	30,300	32,900	--
20	26,500	32,600	--
25	19,600	31,800	--
30	12,900	30,700	32,700
35	9,200	--	--
40	7,100	26,200	31,650
50	--	18,600	29,750
60	--	13,000	21,650
70	--	--	16,600
80	--	--	13,100

TABLE C-II 10.2# PLATE ($\frac{1}{4}$")

ℓ	b	6"x4"x8.5#		8"x4"x10#		10"x4"x11.5#		12"x4"x14#	
		Hs	Hp	Hs	Hp	Hs	Hp	Hs	Hp
8'	10	-	-	-	-	.246	.474	.678	1.18
	15	1.55	4.62	2.05	6.87	3.80	9.56	5.64	12.6
	20	3.19	11.7	5.38	18.4	7.67	23.8	11.0	30.0
	25	4.15	17.8	9.45	37.9	12.9	47.6	18.6	59.2
	30	5.38	25.9	12.1	54.6	15.5	65.3	23.3	84.1
	40	5.29	30.0	11.8	62.4	15.4	78.1	-	-
12'	15	.366	1.09	.735	2.05	1.30	3.31	2.17	4.91
	20	.942	3.56	2.10	7.36	3.01	9.70	4.65	13.0
	25	2.37	10.6	3.96	16.7	5.58	21.6	8.06	27.2
	30	3.27	17.0	5.23	25.5	7.01	32.1	10.3	39.9
	35	3.42	19.7	5.38	29.0	7.20	36.7	10.5	44.9
	40	3.31	21.1	5.15	30.6	6.92	38.8	10.1	47.7

20.4 lb. plate ($\frac{1}{2}$")

ℓ	b	6"x4" x8.5 lb.		8"x4" x10 lb.		10"x4$\frac{1}{2}$" x11.5 lb.		12"x4" x14 lb.		12"x6$\frac{1}{2}$" x27 lb.		14"x6-3/4" x30 lb.	
		Hs	Hp	Hs	Hp	Hs	Hp	Hs	Hp	Hs	Hp	Hs	Hp
8'	20	-	-	-	-	-	-	0	0	0.39	.995	-	-
	25	-	-	-	-	.492	2.92	1.18	5.34	2.68	7.88	3.67	10.2
	30	-	-	.624	4.33	1.27	8.43	2.22	12.8	5.04	16.6		
	40	1.23	9.94	2.60	20.4	3.92	30.2	6.03	40.2	12.8	49.2	15.9	57.6
	50	3.04	26.1	5.12	42.8	7.24	60.7	10.7	77.3	22.3	93.1	27.6	1.09
	60	3.89	34.4	6.21	54.2	8.59	75.1	12.7	95.4	26.1	114	32.2	134
12'	20												
	25							0.14	0.75	0.72	2.20	1.14	3.33
	30					0.17	1.27	0.66	4.05	1.87	6.65	2.0	6.8
	40			0.55	4.8	1.42	12.3	2.38	17.8	5.35	22.8	6.22	25.5
	50			1.88	18.1	2.96	28.8	4.57	38.6	9.75	47.0	12.2	56.5
	60			2.42	26.0	3.64	39.2	5.45	51.5	11.4	62.0	14.2	75.3
16'	20												
	25												
	30							0.11	0.66	0.3	1.09	1.17	4.0
	40					0.49	4.4	1.11	8.65	2.9	13.0	3.64	15.7
	50					1.44	15.2	2.44	22.2	5.1	26.7	6.5	33.4
	60					1.86	21.8	2.97	30.8	6.2	36.8	7.7	46.0

30.6 lb. plate (3/4")

ℓ	b	10"x4" x11.5 lb. Hs	10"x4" x11.5 lb. Hp	12"x4" x14 lb. Hs	12"x4" x14 lb. Hp	12"x6½" x27 lb. Hs	12"x6½" x27 lb. Hp	14"x6-3/4" x30 lb. Hs	14"x6-3/4" x30 lb. Hp	16"x7" x36 lbs. Hs	16"x7" x36 lbs. Hp	18"x7½" x50 lb. Hs	18"x7½" x50 lb. Hp
8'	30	-	-	-	-	-	-	0	0	.222	.82	1.04	2.94
	40	.026	.225	.455	3.8	1.78	8.7	2.43	12.2	3.6	15.0	6.02	20.0
	50	.926	8.65	1.72	15.5	4.32	23.1	5.55	28.4	8.15	37.2	12.6	45.9
	60	2.05	19.8	3.2	29.6	7.15	39.3	9.2	49.5	13.1	63.0	20.1	76.5
	70	3.91	38.2	5.7	54.0	12.3	69.0	15.5	84.5	21.6	106	33.0	127
	80	5.26	52.6	7.55	72.0	15.9	90.0	19.9	109	27.6	136	42.0	164
12'	30											0	0
	40					.24	1.35			1.16	5.45	2.33	8.7
	50			.265	2.72	1.43	9.25	2.04	12.6	3.12	16.8	5.3	22.6
	60	.32	3.64	1.05	11.8	2.81	20.3	3.88	26.3	5.55	33.6	8.8	42.3
	70	1.24	14.5	2.18	25.6	5.2	38.8	6.69	47.5	9.55	60.2	14.6	74.0
	80	1.97	23.8	3.10	38.0	6.85	53.6	8.66	65.0	12.5	82.5	13.9	102
16'	30												
	40									.308	2.30	.93	5.65
	50					.38	3.10	.85	6.78	1.53	10.6	2.60	15.2
	60					1.15	8.7	1.84	13.7	2.86	18.5	4.55	25.0
	70					2.58	18.9	3.46	24.8	5.25	32.6	7.9	42.2
	80					3.58	25.3	4.7	32.6	7.0	42.0	10.2	52.5

TABLE C-III

S = effective amount of plating (in.)

t	S	6"x4"x8.5 lb.			8"x4"x10 lb.			10"x4"x11.5 lb.			12"x4"x14 lb.		
		Zp	Zf	r	Zp	Zf	r	Zp	Zf	r	Zp	Zf	r
	10	13.0	5.7	2.4	19.0	9.0	3.2	24.0	12.4	4.0	30.0	17.4	4.8
	15	18.4	6.1	2.3	26.5	9.1	3.1	33.5	12.8	3.8	41.3	18.0	4.7
¼"	20	24.0	6.2	2.1	34.0	9.2	2.9	42.3	12.7	3.6	52.0	18.2	4.5
	25	29.0	6.2	2.0	41.5	9.2	2.7	51.4	12.6	3.4	63.5	18.0	4.3
	30	34.0	6.1	1.8	48.5	9.1	2.6	60.5	12.4	3.2	79.2	18.1	4.1
	40	43.5	6.0	1.6	61.7	9.0	2.3	78.0	12.6	3.0	95.5	18.5	3.8

t	S	6"x4"x8.5 lb.			8"x4"x10 lb.			10"x4"x11.5 lb.		
		Zp	Zf	r	Zp	Zf	r	Zp	Zf	r
	20	38.0	6.5	1.8	53.0	10.0	2.4	72.0	13.5	3.1
	25	46.0	6.5	1.6	68.0	10.1	2.2	88.0	13.6	2.9
1/2"	30	52.0	6.5	1.5	78.0	10.2	2.1	102	13.7	2.7
	40	61.0	6.5	1.4	95.0	10.0	1.8	129	13.5	2.4
	50	70.0	6.5	1.3	110	10.0	1.7	152	13.5	2.2
	60	79.0	6.5	1.3	124	10.0	1.5	169	13.6	2.0

t	S	12"x4"x14 lb.			12"x6½"x27 lb.			14"x6-3/4"x30 lb.		
	20	90.0	19.5	3.8	104	39.8	4.9	123	49.5	5.6
	25	110	19.6	3.6	129	40.2	4.6	150	49.9	5.3
1/2"	30	128	19.7	3.4	152	40.5	4.4	176	50.0	5.1
	40	161	19.7	3.1	192	40.5	4.0	227	49.8	4.8
	50	197	19.6	2.8	229	40.4	3.8	278	49.4	4.4
	60	224	19.5	2.6	264	40.5	3.5	328	49.3	4.1

t	S	10"x4"x11.5 lb.			12"x4"x14 lb.			12"x6½"x27 lb.		
	30	120	14.5	2.3	163	20.1	3.0	200	41.3	4.0
	40	150	14.5	2.1	202	20.2	2.7	258	41.5	3.5
3/4"	50	170	14.6	1.9	236	20.3	2.3	309	41.7	3.2
	60	186	14.5	1.7	264	20.4	2.2	354	41.4	2.9
	70	200	14.5	1.6	287	20.5	2.0	392	41.0	2.8
	80	212	14.4	1.5	306	20.6	1.9	428	40.8	2.7

t	S	14"x6-3/4"x30 lb.			16"x7"x36 lb.			18"x7½"x50 lb.		
	30	245	51.3	4.7	287	69.5	5.5	345	102	6.8
	40	306	52.0	4.2	370	70.5	5.1	442	102	6.4
3/4"	50	365	51.8	3.9	455	71.8	4.8	540	102	5.9
	60	420	51.3	3.6	537	73.0	4.5	640	102	5.6
	70	470	51.0	3.3	616	74.5	4.3	736	102	5.2
	80	520	50.8	3.2	690	75.5	4.1	826	102	5.0

Appendix D Sample Calculation

$$H = \frac{33{,}000}{a^2} \quad \frac{8 \times 144\, Z}{64\, b} \left[1 - \frac{\sigma_{cr\,p}}{\sigma_{cr\,col.}} \right]$$

for $a = 96''$

$$H = 64.45 \frac{Z}{b} \left[1 - \frac{\sigma_{cr\,p}}{\sigma_{cr\,col.}} \right]$$

1. Select $t = \frac{1}{2}''$
2. Select $10'' \times 4'' \times 11.5^{\#}$ Stiffener. $A_s = 2.63\ in^2$
3. Select $b = 30''$
4. Compute $b/l = b/a = \frac{30}{96} = 0.312$
5. Enter curve of effective breadth with b/l; Read off $b_e/b = 0.86$
6. Compute $b_e = b \times b_e/b = 30 \times 0.86 = 25.8$
7. Enter Curves of Section Modulus vs. effective breadth with $b_e = 25.8''$ and read off Z_p and Z_f

$$Z_p = 90.8\ in^3$$
$$Z_f = 13.7\ in^3$$

8. Enter curves of radius of gyration vs spacing with $b = 30''$ and read off r

$$r = 2.67\ in.$$

9. Compute $a/r = 96/2.67 = 35.9$
10. Enter curve of column critical stress vs. a/r and read off $\sigma_{cr\,col}$

$$\sigma_{cr\,col} = 32{,}150\ p.s.i.$$

11. Enter curve of plate critical stress vs. spacing with $b = 30''$ (for $t = \frac{1}{2}''$) and read off $\sigma_{cr\,p}$.

$$\sigma_{cr\,p} = 30{,}700\ p.s.i.$$

12. Compute H_F and H_P

$$H_F = \frac{64.45}{30} \times 13.7 \left[1 - \frac{30{,}700}{32{,}150} \right] = 1.272\ ft.$$

$$H_P = H_F \times \frac{Z_P}{Z_F} = 1.272 \times \frac{90.8}{13.7} = 8.43\ ft.$$

13. $w = (A_s + bt)\frac{\gamma}{b} = (2.63 + 15.0) \times \frac{.283}{30} = 0.1660 \ lb/in^2$

14. Load, $N = \frac{\sigma_{cr\,p}}{b} \times (A_s + bt) = \frac{30,700}{30}(2.63 + 15.0)$

$$= 18,050 \ lb/in$$

15. Tension in flange when H_p is acting:

$$\sigma = -\sigma_{cr\,p} + \frac{M}{Z_F}$$

$$= -\sigma_{cr\,p} + \frac{\gamma H_p b a^2}{8 \times 144 \times Z_F}$$

$$= -30,700 + \frac{64 \times 8.43 \times 30 \times (96)^2}{8 \times 144 \times 13.7}$$

$$= -30,700 + 9430$$

$$= 21,270 \text{ p.s.i. tension (O.K.)}$$

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